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On social and psychological aspects of a negligible reception of Natanson's article of 1911 in the early history of quantum statistics

Abstract

Possible reasons are studied why Ladislas (Władysław) Natanson's paper on the statistical theory of radiation, published in 1911 both in English and in the German translation, was not cited properly in the early history of quantum statistics by outstanding scientists, such as Arnold Sommerfeld, Paul Ehrenfest, Satyendra Nath Bose and Albert Einstein.

The social and psychological aspects are discussed as background to many so far discussions on the academic evaluation of his theory.

In order to avoid in the future such Natansonian cases of very limited reception of valuable scientific works, it is proposed to introduce a digital tag in which all the information of

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O społecznych i psychologicznych aspektach znikomej recepcji artykułu Władysława Natansona z 1911 roku we wczesnej historii statystyki kwantowej

Abstrakt

Dyskutowane są możliwe przyczyny, które sprawiły, że artykuł Władysława Natansona na temat statystycznej teorii promieniowania, opublikowany w 1911 r. zarówno w języku angielskim, jak i w tłumaczeniu na język niemiecki, nie był prawidłowo cytowany we wczesnej historii statystyki kwantowej przez wybitnych naukowców, takich jak Arnold Sommerfeld, Paul Ehrenfest, Satyendra Nath Bose i Albert Einstein.

Omówiono aspekty społeczne i psychologiczne, które pozwalają lepiej poznać tło wielu dotychczasowych dyskusji na temat oceny jego teorii.

Aby uniknąć w przyszłości takich natansonowskich przypadków bardzo ograniczonej recepcji wartościowych publikacji naukowych, proponuje się wprowadzenie cyfrowego znacznika, w którym wszystkie dotychczasowe informacje o odpowiednich publikacjach powinny być automatycznie gromadzone i aktualizowane.

Słowa kluczowe: *nierozróżnialność stanów kwantowych, historia statystyki kwantowej, Władysław Natanson, Max Planck, Arnold Sommerfeld, Paul Ehrenfest, Satyendra Nath Bose, Albert Einstein, Jun Ishiwara, cytowanie*

1. Introduction

Indistinguishability is the most basic concept underlying the difference between quanta and classical particles. The concept was first recognized and described by Ladislas (Władysław) Natanson in his paper

published in *Bulletin de l'Academie des Sciences de Cracovie*, *Classe des Sciences Mathématiques et Naturelles. Serie A*: *Sciences Mathématiques* on March 6, 1911 (Wł. Natanson 1911a). In §2 of this paper, "Indistinguishability" is expressed as follows:

> […] the elements or units of energy are all regarded as being undistinguishably alike.

The paper was originally written in English (Wł. Natanson 1911a), but then was translated into German and published also in *Physikalische Zeitschrift* on August 15, 1911 (Wł. Natanson 1911b; for more see section 2, below). Natanson proposed a new idea to derive Planck's radiation law without using the hypothetical interpretation proposed by Max Planck in 1900. His method was somewhat different from the well-known one proposed by Satyendra Nath Bose, which today is called Bose-Einstein statistics. The differences between Natanson's and Bose's approaches have been discussed by many researchers such as Oliver Darrigol (1991), Bogdan Lange (1997), Jagdish Mehra and Helmut Rechenberg (2001), and Józef Spałek (2005).

Bose's paper was published in *Zeitschrift für Physik* on July 2, 1924 (Bose 1924). An interesting story was widely circulated regarding the process of its publication. In an essay written in 1965 (Bose 1965), Bose recounted his situation during that time in a following manner:

> I wrote it in English and sent it to England for publication. Meanwhile, I was also curious to know Einstein's opinion of the new idea, so I also sent a copy of the unpublished paper to him. I did not hope for any unexpected result. In fact, I had very little hope that I would be able to draw his attention to this matter at all.

Contrary to Bose's expectation, Albert Einstein was deeply impressed by his idea and immediately translated it into German himself and published it in *Zeitschrift für Physik*. Inspired by Bose's idea, he published a series of papers in quick succession concerning the evolution of the Bose's idea to ideal gas resulting the prediction of the condensation phenomenon in ideal gas. The first of these was published in *Sitzungsberichte der Preussischen Akademie der Wissenschaften* on July 10, 1924 (Einstein 1924). The date was just eight days after Bose's first paper was published.

The 22 years after World War II, Friedrich Hund (1967) noticed:

> The Bose statistics of light quanta was […] the same as that earlier applied by Planck for energy quanta (Chapter 2) and thus led to the Planck radiation formula. This method of counting events for indistinguishable particles, which had already been perfectly clearly recognized by Natanson in 1911, was subsequently to be called Bose statistics (Natanson's work had of course been forgotten by 1924) (quoted from Hund 1974, p. 145)

Alfred Kastler had a similar opinion (Kastler 1983). This is confirmed by an essay written by Wojciech Natanson (2012), a son of Władysław. He wrote:

> Nieco wcześniej wielki francuski uczony alzacko-szwajcarskiego pochodzenia, laureat Nagrody Nobla i poeta, prof. Alfred Kastler przysłał mi broszurę, w której wykazywał, że ojciec swymi badaniami wyprzedził pewne odkrycia Einsteina i Plancka, którzy o tym... nie wspomnieli, chociaż z Władysławem Natansonem korespondowali i znali jego prace (Woj. Natanson 2012, p. 42).

This can be translated as:

> A little earlier the great French scholar of Alsatian-Swiss origin, a Nobel laureate and a poet, Prof. Alfred Kastler, sent me a brochure which showed that my father preceded with his research some discoveries of Einstein and Planck, who did not mention this, although they corresponded with Władysław Natanson and knew his paper[19].

Why did Bose and Einstein not refer to Natanson's paper in their papers?

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[19] The translation of Polish text into English was provided by Professor Piotr Petelenz of the Jagiellonian University.

2. A great opportunity

Soon after Natanson had published his paper in the journal of the *Academy of Arts and Sciences* in Kraków, he received two short business letters from Friedrich Krüger, the chief editor of *Physikalische Zeitschrift*; these letters are included in the collection of Natanson's correspondence at the Jagiellonian Library. In the first letter, dated April 24, 1911 (Archive 2, p.117), the editor asked for Natanson's permission to translate his paper into German, which led to its publication in *Physikalische Zeitschrift*. At the beginning of the letter, Krüger wrote:

> Thank you very much for sending us a reprint of your very interesting paper regarding the statistical theory of radiation.

This confirms that Natanson sent the reprint of his paper to Krüger, and we can deduce that Krüger read it and recognized that it was worth publishing. This means that Natanson had an opportunity to inform influential scientists in Germany about his idea by publishing it in a well-established journal of good circulation. In addition, he might have had another nice opportunity.

That summer, the 11th Congress for Polish Physicians and Natural Scientists had been scheduled in Kraków (we will hereafter call this the Kraków Congress), and Einstein, who was in Prague, was invited to join. Since Natanson was the director of the Department of Theoretical Physics of the Jagiellonian University and he was very active in the Academy of Arts and Sciences in Kraków, he would have been a member of the local committee of the Congress. It is probable that he recommended inviting Einstein to the Congress. Einstein's theory of relativity was a fascinating subject that was attracting attention at the time, and many scientists and philosophers in Kraków would have looked forward to an opportunity to listen to Einstein's lecture. It would have been considered a premier event that would add prestige to the Congress, which was held from July 18 to 22[20] as scheduled and took place about three months before the First Solvay Congress, which was held in Brussels, Belgium.

[20] And so it happened – see Polak 2016, pp. 251–253.

The Solvay Congresses are now considered to have been the most influential academic workshops on the development of quantum mechanics. The main theme of the First Solvay Congress was the theory of radiation and quanta, the very topic of Natanson's paper. About twenty leading scientists from western European countries – including Max Planck, Arnold Sommerfeld, Maria Curie-Skłodowska and Albert Einstein – were invited to attend and a formal letter of invitation, dated June 9, was sent to each of them from Ernest Solvay, sponsor of the Congress. If Natanson were aware of these events, he would have hoped that Einstein's visit to the Kraków Congress would give him a chance to discuss his statistical theory of light with Einstein before the Solvay Congress.

The second letter, dated July 22, 1911 (Archive 2, p. 118), is a reply from Krüger to Natanson's inquiry about the scheduled publication date of his paper. The letter's date corresponds to the closing day of the Kraków Congress. Natanson most likely expected his paper to appear before the Kraków Congress, but since he did not receive any notice from the publisher even after the Congress had begun, he might have become impatient. Krüger replied that the proofed draft had been accepted, but that the publication was scheduled in August because there were many other papers whose publication was more urgent. Therefore, the German version of Natanson's paper was not published in time for the Kraków Congress.

There is a draft of Einstein's letter to the Department of Physics of the Jagiellonian University that must have been sent from Prague before July 21, 1911 (Archive 1, vol. 5, Doc. 273). In this draft, he notified the department – i.e. Natanson, the director of the department – that he would not attend the Congress. However, no reason for this is given in the draft. As a result, Natanson could not meet Einstein at the Congress and lost his chance to explain his new idea to Einstein.

This is confirmed by another interesting letter dated September 18, 1911 (Archive 1, vol. 5, Doc. 285), sent by Einstein's mother, Pauline to her son, Albert. She was living at Heilbronn, located about 50 km north east of Karlsruhe. Einstein had been staying in the German part of Prague's Charles University since April, 1911. In the letter, she asked Albert to stop at her house on the way to Karlsruhe from Kraków because she wished to see him. One of the foot-notes given to the letter in this book reads:

> Einstein had apparently told his mother that he planned to attend the eleventh congress of Polish physicians and natural scientists in Cracow, but neglected to inform her that it had already taken place in July and that he had declined the invitation (Archive 1, Doc. 273).

3. An influential comment

Who did read Natanson's paper? We can identify at least three persons who probably read it. One is Max Planck, who provided a foot-note on Natanson's paper in the Proceedings of the First Solvay Congress (Planck, 1911a). This footnote provides clear evidence that he had read Natanson's paper and recognized that it was worth quoting. This is the only instance that we can find of a third party referring to Natanson's paper before World War II. The footnote, however, might have had an unexpectedly negative impact on Natanson and his work. Planck noted:

> These calculations are complete and do not contain such uncertainty that recently Natanson described in *Phys. Zeitschr.* (Planck 1911a).

The question remains: why did not Planck refer to Natanson's paper in his text but in a footnote? Oliver Darrigol (1991) gives us the answer in a detailed discussion in his article, "The early symptom of indistinguishability and holism". He showed that Planck added the footnote after a discussion with Paul Ehrenfest. Such background to Planck's footnote is deeply relevant to the second candidate, Arnold Sommerfeld.

According to the Jagiellonian Library's collection of Natanson's correspondence, Sommerfeld sent a letter dated October 3, 1911 to Natanson from Munich (Archive 2, p. 107), where he wrote:

> [...] Ich bin Ihnen aufrichtig dankbar, Dass Sie mir regelmässig Ihre sehr interessanten Arbeiten zusenden, die ich stets genau verfolge; ich werde mich bald mit meinem Carlsruher Vortrag über Quantentheorie und einigen Anderen revangieren. […]

This can be translated as:

> […] I sincerely appreciate that you regularly send me your very interesting works, which I always thoroughly follow; I will soon return the favour with my Karlsruhe lecture on quantum theory, and some other papers. […]

In Karlsruhe, the 83rd meeting of the German Natural Scientists and Medical Doctors Association was held from September 24 to 29, 1911, and there Sommerfeld spoke on "Planck's action quanta and its importance in molecular physics." His talk was reported as a lengthy paper (Sommerfeld 1911) in *Physikalische Zeitschrift,* the same journal in which the Natanson's paper had been published. Since it was only Natanson that had published his paper on the statistical theory of radiation during the relevant period, I speculated that he must have sent a reprint of the paper to Sommerfeld before that meeting. Probably, Sommerfeld replied from Munich after coming back from the meeting.

A letter dated October 16, 1911 and sent from St.Petersburg by Ehrenfest to Sommerfeld (Archive 3, München, DM: Archiv HS 1977--28/A, 76) clearly shows that Ehrenfest and Sommerfeld read Natanson's paper. In this letter, Ehrenfest explained his opinion about the difference between Planck's and Einstein's hypotheses on the energy quantum and mentioned Natanson's paper. Ehrenfest wrote:

> Die Bemerkungen, die kürzlich Nathanson über die combinatorischen Grundlagen der Planckschen Theorie publiziert hatte ich ebenfalls gefunden und vor dem Erscheinen der Arbeit von Nathanson in der hiesigen phyikalischen Gesellschaft vorgetragen. Aber Nathanson hat die Lösung der Schweierigkeit nicht gefunden: er hat eben nicht bemerkt, daß die Planckschen und Einsteinsche Hypothese total verschieden sind.

This can be translated as:

> Remark: I had also found the recent publication of Nathanson on the combinatorial foundations of Planck's theory. I had presented (the idea) at the local physical society before Nathanson's paper was published. But Nathanson

> did not find the solution to the difficulty: he did not notice that Planck's and Einstein's hypotheses are totally different.

The important thing here is not his opinion about Natanson's paper, but that he specifically acknowledged being aware of it. The word "ebenfalls" shows that Ehrenfest and Sommerfeld had read the Natanson's paper.

I considered that Ehrenfest's opinion about Natanson's paper might have been shared by outstanding scientists who attended the First Solvay Congress. Probably, their interest was not on the conceptual importance of the indistinguishability of quantum states, regarding it as an *a priori* hypothesis. Anyways, Planck's footnote must have sent a strong message by which many readers of the proceedings of the Solvay Congress might have lost interest in Natanson's paper. This is one of the reasons why Natanson's paper was not cited in many articles.

4. Japanese who may have read Natanson's paper before 1924

There are two Japanese scientists who might have read Natanson's paper of 1911 before 1924: Hantaro Nagaoka and Jun Ishiwara.

Hantaro Nagaoka was a Japanese scientist who proposed a planetary model of the atom in which a positively charged center is surrounded by a number of revolving electrons in 1904. The "Hantaro Nagaoka Papers" in the collection of the National Museum of Nature and Science (Archive 4) hold reprints of six versions of Natanson's papers published between 1904 and 1931. All of them are reprints from journals of Kraków. This suggests that they were provided by Natanson and the paper in question (Wł. Natanson 1911a) is one of them. Therefore, it is highly probable that Nagaoka would have read it. He visited Berlin in 1893 to study under Planck and stayed in Germany and Austria for three years. He later visited Europe several times until 1935. He was very influential in the Japanese science and scientific technology community, but any evidence that he was interested in Natanson's and Bose's papers on statistical properties of light have not been found so far. The reason may have been that his interests were focused on atom structure and atomic spectroscopy and on his application-oriented researches later on (Okamoto 2006).

Jun Ishiwara was a student of Nagaoka at Tokyo Imperial University.[21] He was appointed associate professor of Tohoku Imperial University in Sendai in April 1911, just after the university was opened to the public. In 1912 he published the paper titled "Beiträge zur Theorie der Lichtquanten" in *Science Reports of the Tohoku Imperial University* (Ishiwara 1911–1912), received for publication on October 10, 1911. In this paper, he derived Planck's radiation law by assuming that the unit volume of the phase space should be h^3 where h is the Planck's constant. He also took account of the degree of freedom, 2, of radiation field due to its polarization in his calculation. Therefore, his approach was almost the same as Bose's. However, he did not refer to Natanson's paper of 1911. There are two plausible reasons: One is that Nagaoka might not have shown him Natanson's original paper, which he had as mentioned above, and the other is that he could not have read Natanson's paper in *Physikalische Zeitschrift*: the issue of its publication, was certified to be registered on May 17, 1912 by the library of his university. I speculated that if Ishiwara had a chance to communicate with Natanson through Nagaoka, the science community's awareness of these scientists and even the history of quantum statistics might have somehow been altered.

Ishiwara lived in Europe from March 1912 to April 1914. He studied under Sommerfeld in Munich during the summer of 1912. He published another paper with similar content to the previous one (Ishiwara 1911–1912) in *Physikalische Zeitschrift* on September 4, 1912 (Ishiwara 1912; Darrigol 1991), but he did not refer to Natanson's paper of 1911. In 1939, Ishiwara wrote an essay, titled "Memory of Professor Sommerfeld" (Ishiwara 1939; original text in Japanese). I found an interesting passage by which we can suppose the reason. It can be translated as:

> Since I was interested in the problem of light at that time, I tried some theoretical calculations assuming that the light-quantum is particle-like. When I asked Professor Sommerfeld for his opinion, he told me that the idea may be better, but he did not appear to accept it with any certainty. As I mentioned above, he strongly believed that

[21] Describing the activities of Jun Ishiwara, I follow in the footsteps of Seiya Abiko (2000).

> since the wave-nature of X-rays had been verified by the observation of Laue's spots, the character of the discontinuity of radiation should not be due to its inherent nature but should be due to some atomic mechanisms that emit it. I think that his opinion was natural at that time. Nevertheless, when I showed him my mathematical method based on a statistical approach to light-quanta, he sometimes told me that it seemed interesting.

Ishiwara's opinion is verified by reading his diary (Archive 5, #1041) that he wrote during his stay at Sommerfeld's laboratory in Munich. Some relevant parts can be translated as:

> May 7, 1912, Tuesday: It's raining, as usual.
>
> I visited Professor Sommerfeld in the morning. When I showed him my paper, he told me that it was interesting. He asked me some questions, and also asked me whether I had come up with a topic to study. He recommended me to study "Lichtquanten" saying that it is currently a big topic. He also kindly recommended me to join tomorrow's colloquium. I went home with thankful mind to him.

Here, the "my paper" that he is referring to would have been one published in Sendai (Ishiwara, 1911).

> May 24, Friday: It's raining today, too.
>
> Because Professor Sommerfeld gave his lecture after Mr. Laue's one, I did not go to the Berlitz. Sommerfled asked me if I had any comments on Laue's lecture of "Elektronen Theorie" because Laue discussed my paper. I could not respond properly because I had forgotten it unfortunately. I reconsidered it after I came back home, but my opinion seemed better. Sommerfeld also asked me whether I still believed "Lichtquanten" as the meaning of "diskontinuierliche Struktur". And he added that he could not believe the idea, showing me Röntgen-ray photos. I don't like to insist on my opinion in neglect of such experimental facts, but I feel that the hypothesis of "Quantenstruktur" has more advantages notwithstanding it has a little disadvantage when it comes to explaining "Resonance" and "Serienstruktur".

From these diary entries, we can suppose that Sommerfeld might have not informed him of Natanson's approach. Ishiwara also visited Zurich to meet Einstein in 1913 and became acquainted, but there is no evidence that he discussed his idea with Einstein.

After he returned home to Sendai, he was promoted to professor of theoretical physics of Tohoku Imperial University in 1914. He was also active as a poet, belonging to a famous poetry circle in Japan. About 1920, he had a love affair with another poet in the circle, a scandal during the day, and according to Yumiko Mori, his granddaughter, he faced serious bashing by mass media. Subsequently, he resigned from the university in August 1921. His academic research in physics came to a close, but his activity as a poet and writer continued.

Ishiwara is remembered as having extensively contributed to the enlightenment of the Japanese about modern physics and Einstein's theory of relativity. There is no evidence, however, that he was interested in Natanson's paper later on after he had returned home. If he had more chances to know of Natanson's activity in various literary fields in addition to physics during his stay in Europe and described them in his essays, someone might have recognized the importance of Natanson's paper of 1911 when the Bose's paper was published in 1924.

5. How about Einstein?

According to Bose's essay referred to in the Introduction, he first submitted his original manuscript to English journals such as *Philosophical Magazine* and also sent it to Einstein with his letter. Bose wrote:

> […] I do not know sufficient German to translate the paper. If you think the paper worth publishing, I would be grateful if you arrange for its publication in *Zeitschrift für Physik*. […] (Harum ar Rashid 1995).

This letter can be read as Bose asking for Einstein to translate it in German and to submit it to a publisher. However, it is also clear Bose did not explicitly ask Einstein to do the translation. Anyway, Einstein translated it by himself and arranged to have it published in *Zeitschrift für Physik* quickly, as Bose hoped. Immediately afterwards, he published his own paper on the subject in *Sitzungsberichte der Preussichen Akademie* (Einstein 1924), where he of course, referred to Bose's paper.

In the case of Natanson's original paper of 1911 written in English (Wł. Natanson 1911a), it was translated into German by Max Ikle. However, during World War I, there were implicit regulations as to when and whether German publishers could accept English manuscripts written by non-German authors (Wolff 2003). After the war, the regulations were formally relaxed. In fact, a paper by R.N. Ghosh in 1925 (Ghosh 1925), was even published in *Zeitschrift für Physik* in English. Therefore, I speculated that Bose knew that Karl Scheel, the editor-in-chief, could arrange a translation of Bose's manuscript written in English into German, if his paper was accepted. In such case, Bose did not need to ask Einstein for a translation.

Einstein sent Bose a post card dated July 2, 1924 (Harum ar Rashid 1995) informing him the paper had been sent to the publisher, as Bose wanted. In fact, the paper was accepted. However, Einstein added a strange note at the end of the paper (Bose 1924; the image file of the original manuscript is Archive 1, 1-045). I thought that this somewhat unusual interpretation is helpful to understand the Einstein's situation immediately following:

> Anmerkung des Üebersetzers: Boses Ableitung der Planckschen Formel bedeutet nach meiner Meinung einen wichtgen Fortschritt. Die hier benutzte Methode liefert auch die Quantentheorie des idealen Gases, wie ich an anderer Stelle ausführen will.

Max L. H Delbruck (1980) translated it into English with his critical comments as follows:

> There is evidence of haste in Einstein's handling of the paper as Bose is not credited with his initials. Also, the paper is astoundingly brief and abrupt. It has no literature references. I have a strong suspicion that Einstein cut it short, perhaps even rewrote it. At the end of this four page paper there is a highly unusual footnote, a kind of thunderbolt that says, "Bose's derivation of Planck's law in my opinion constitutes an important step forward. The method here employed also yields the quantum theory of the ideal gas, as I will show later." Who would not like to have such a footnote to his paper!

Why was Einstein in such a hurry? I think that a hint of the answer is speculated from Bose's situation described by Jagdish Mehra and Helmut Rechenberg (1982, p. 565):

> In the beginning of June 1924 he had the manuscript of a paper ready, which he entitled "Planck's Law and the Light Quantum Hypothesis. Since he had the impression that his treatment very much followed Einstein's thought, indeed, it completed the proof which Einstein had been searching for many years, Bose decided to send his paper to Einstein […]

If I may be so bold, I speculated that Einstein was a little panicked after reading Bose's manuscript and felt as if he had been upstaged by a young unknown. Such a psychological reaction would be familiar to any researcher who unexpectedly finds out that someone else is going to propose new or competitive ideas. The reason is that because it is the evidence that the topic is surely state of the art. He must, therefore, have wanted to publish Bose's paper and then as soon as possible publish his own paper[22]. Under such circumstances, I cannot think of Einstein quoting Nathanson's paper.

When Einstein finally met Natanson, he sent a letter, dated 12 February 1915 (Archive 1, vol. 8, Doc. 56), to his intimate friend, Michele Besso. He wrote:

> Presently, Natanson (theorist of physics) of Lvov and his fellow kinsman, to whom I have become very fond, is here.

However, I could not find clear evidence that Einstein and Natanson exchanged their opinions on the quantum statistics.

6. Effects of complex emotions in war time

Even if Einstein had forgotten or ignored Natanson's paper when he read Bose's manuscript in 1924, could someone else who knew the Natanson's paper of 1911 have reminded him of it? Sommerfeld would

[22] A quiz for students at your lecture: As an unlikely story, if Einstein ignored the first Bose's letter with the original manuscript and published his papers without referring Bose's idea, how could Bose determine whether his idea was stolen by Einstein?

have been the most probable candidate. As mentioned in section 4, he was skeptical about the light quantum in 1912. However, when Bose's and Einstein's papers were published in 1924, there might have been a chance for him to remind Bose or Einstein about Natanson's approach, as Ehrenfest did with Planck before. However, there is no evidence that he mentioned Natanson's paper. I speculated that a possible reason for Sommerfeld not mentioning the paper was his image of Natanson that would have unconsciously been influenced by the prevailing social atmosphere before World War I. Sommerfeld's experience is not an exception, and everyone would have had similar experience, to some degree or other.

The Germans declared war on Russia on August 1, 1914. Three days later, England declared war against Germany. The English scientific community followed public opinion and opposed Germany. The German scientific community was strongly against them. German nationalists supported their stance (Wolff 2003). Wilhelm Wien sent a letter dated on December 22, 1914 asking Somerfield to sign an appeal against Engländerei[23] (Archive 3, München, DM: Archiv NL 89, 059). Sommerfeld sent a letter on December 25, 1914 to Wien and accepted Wien's request (Archive 3, Berlin, SBPK: Autogr.I/1253). In this way, Sommerfeld was at the center of such a toxic social atmosphere. After Germany was defeated in World War I and the Treaty of Versailles was signed in June, 1919, tensions between German and English scientific communities were relaxed on the surface, but in Germany an unendurable discontent[24] remained.

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[23] One of the origins of such movements in the German scientific community was their complaint against English scientists who did not cite their papers properly (Wolff 2003). A similar sentiment was shared by Nagaoka, when he published his paper on the structure of the atom. He wrote in his essay: (When Ernest Rutherford succeeded in verifying the presence of a nucleus at the center of an atom experimentally, seven years after Nagaoka published his paper in *Philosophical Magazine*) "Rutherford wrote to me saying «I had not read your paper on your atomic model yet». Since he had published all of his papers in that journal, it was difficult for me to understand this. He should have been familiar with that journal for the last seven years. His excuse does not ring true. In addition, his model was essentially similar to my Saturn-like model. My sentiments will be left to the wise reader's guess" (the original text was written in Japanese: Nagaoka 1950).

[24] On July 20, 1925, Max Theodor Felix von Laue sent Sommerfeld a letter that suggested a serious conflict had broken out within the German-speaking physics com-

On the other hand, Poland reestablished her independence after the war, and Polish scientific community was to be integrated within the framework of the victorious countries. Natanson worked actively in the Polish scientific community. In 1919, the Polish Academy of Arts and Sciences, as representative of the Polish State, sent a delegation to Brussels to take part in a conference aimed at establishing scientific collaboration between allied states, and that would form the International Research Council (Klecki 1939).

According to Natanson's biography (Wł. Natanson 1958, p. 119), he kept a politically neutral stance. However, even if he believed so, he might still have been regarded as disloyal or untrustworthy by German scientists. Under these social circumstances, I suspected that a biased image of Natanson had somehow developed in Sommerfeld's mind. William A. Blanpied described that the social atmosphere at that time gave Bose a strong psychological influence:

> Bose visited France for the second time in 1951, and for the next several years traveled extensively. But he never had any desire to see Germany again. The memories of 1926 when young German girls confided that their sons would revenge the "betrayal of 1918" remained too strong for him (Blanpied 1972, p. 1217).

Sommerfeld sent a letter dated November 1, 1919 (Archive 3: Private property, Warsaw) to a Polish physicist Adalbert Rubinowicz, who knew and corresponded with Natanson, and a former research assistant

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munity (Archives 3, München, DM: Archiv HS 1977-28/A, 197). In this letter, one finds: "Als die zu so trauriger Berühmtheit gelangte Arbeit von Bose englisch erschienen war, hat sich nicht nur Lenard, sondern auch andere Mitglieder der Deutschen Physikalischen Gesellschaft mit Beschwerden darüber an M.Wien gewandt. (When Bose's work appeared in English and became the subject of unfavorable rumors, Lenard as well as other members of the German Physical Society lodged their complaints with M.Wien.) The „Bose" in this letter has been identified as S.N.Bose by the Sommerfeld project. This suggests that the conflict was caused by Bose's paper of 1924. Two opinions have been proposed concerning „Bose": Stefan L.Wolff (2003) implied that „Bose" in fact was R.N. Ghosh (1925), indicating that Laue was mistaken. However, Rajinder Singh (2001) accepted Laue's description as it was. I think these opinions are not alternative, but rather correlated. Further study is necessary to learn why Laue referred to „Bose".

of Sommerfeld. This letter was his reply to Rubinowicz's letter asking for Sommerfeld's help in finding an academic post. Sommerfeld replied:

> It would be very difficult to find positions in Poland. Natanson is not reliable because his activity is limited within his domestic community. You should contact M. Curie. She is not poisoned by chauvinism.

The reference to 'chauvinism' in his letter suggests his state of mind. Even if he remembered the Natanson's paper when Bose published his paper in 1924, he might have avoided bringing it to Bose's or Einstein's attention. This may be another reason for Natanson's paper being forgotten within the physics community.

7. The View from London

Half a century passed before Hund's book was published in German in 1964 (and its translation into English in 1974). Was there any possibility that someone else had recognized the importance of Natanson's 1911 paper during this period?

Inspec is a well-known data base; before it was established in 1967, a specialized English journal, *Science Abstracts*, had played its role since 1898.[25] In *Science Abstracts*, selected abstractors from professional research fields read all the relevant papers published in available journals and published their comments on the respective papers. Thanks to this journal, we can trace almost all activity of Natanson in physics[26].

In *Science Abstracts*, Samuel Hawksley Burbury was the main writer of the abstracts of the Natanson's papers concerning thermodynamics and fluid mechanics. He was a professor at the University of Cambridge and 30 years older than Natanson. His main work was as a lawyer, but he was also a famous scientist in the field of statistical physics.[27] Their correspondence shows that Burbury and Natanson were intimate acquaintances.[28]

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[25] Cf. Institution of Engineering and Technology 2018.

[26] His activity in various fields is described by Jan Hulewicz and Tadeusz Piech (1977); Bronisław Średniawa (2007); Michał Kokowski (1993; 1994; 1997; 2009).

[27] Cf. J. L. 1912.

[28] Cf. Archive 2.

For example, in 1903 when Natanson was 39 years old, one of his papers (Wł. Natanson 1901) was seriously criticized by a mathematician, Stanisław Zaremba, a fellow professor at the Jagiellonian University (Zaremba 1903). From the abstract of Zaremba's paper provided by Burbury (Burbury 1904c) in *Science Abstracts,* we can imagine that the discussion between the two professors would have been hard-edged and possibly acrimonious. However, Burbury generously proposed to Natanson a reasonable way to settle this conflict in the abstract.[29]

For Natanson's papers published in 1911 (1911a or 1911b), corresponding reviews by Burbury would normally have appeared in *Physics Abstracts* in 1912. However, Burbury passed away at 80 years of age on August 18, 1911, about two weeks after publication of Natanson's paper in *Physikalische Zeitschrift*. The editorial arrangements for Natanson's paper seem to have been affected by Burbury's death.

Article #344 is identified as an abstract of Natanson's papers on the "Statistical theory of radiation" that were published in *Bulletin de l'Académie des Sciences de Cracovie* on March 6, 1911 (Wł. Natanson 1911a) and *Physikalische Zeitschrift* on August 15, 1911 (Wł. Natanson 1911b) respectively, as mentioned above. However, the abstract is blank. The next article, #345, is an abstract of Planck's article (Planck 1911b), "The Quanta-Emission Hypothesis", which was published after July. Next is article #346, an abstract of Nernst's article (Nernst 1911), "Inconsistency of my Heat Theorem and van der Waals' Equation at very Low Temperatures", which was published on August 28, 1911. These abstracts are also blank.

On the other hand, article #341 is an abstract of Poincaré's paper (Poincaré 1911), "Radiation Quanta", published on December 4, 1911 in *Comptes Rendus*. The abstracter was E. H. Barton, a professor of Nottingham University (Barton 1912). He was the main abstractor for Natanson's papers on optics, and he was also the abstractor of papers published in German journals by Einstein *et al.*

Under the circumstances, Burbury might have seen Natanson's paper published in *Bulletin de l'Academie des Sciences de Cracovie*, but his work

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[29] Burbury was well prepared to this task, since before abstracting Zaremba's paper, he abstracted two other Natanson's papers (1903a; 1903b) on related subjects: Burbury 1904a; 1904b.

might have been delayed for reasons related to his failing health. It is also understandable that the work was not handed over to other reviewers such as Barton for various reasons. In all likelihood, Natanson's paper of 1911 would have been reasonably evaluated if Burbury had been able to write the abstract. This story does not end here.

Barton (1912b) wrote article #733, a long positively evaluated review of a Natanson's paper "Energy Content of Bodies" that was published in *Bulletin de l'Académie des Sciences de Cracovie* in 1912 (Wł. Natanson 1912). He introduced Natanson's arguments that Planck's theory is essential to understanding material at the particle level and that aspects of Planck and Einstein were completely different and would be difficult to reconcile. Since this paper is directly related to the Natanson's paper of 1911 (Wł. Natanson 1911a; 1911b), had Barton reviewed the Natanson's paper his comments probably would have been supportive. In this sense, the criticism Ehrenfest expressed in his letter to Sommerfeld seems misleading.

Time passed, and when Bose's paper was published in 1924 (Bose 1924), its abstractor was not Barton, but S. G. Baker. He extracted the essence of this paper (Baker 1924) and wrote:

> Einstein suggested that Bose's theory is applicable to the theory of an ideal gas and promised that the details would be explained by him in the near future.

Given such an introduction, it is natural that the paper's contents and expectations regarding Einstein's explanation would arouse the attention of many readers. Barton passed away suddenly on September 23, 1925. If he had reviewed Bose's and Einstein's papers, he might have recalled the essence of Natanson's paper when writing the abstracts for *Physics Abstracts*.

Thus, Natanson's paper of 1911 (1911a or 1911b) unfortunately lost many chances to catch the interest of readers of *Physics Abstracts*; the chain of events is almost as if there was a conscious effort to have it escape the abstractor's eyes.

8. To the eyes of a student

One of the pleasures of being a professor at a university is to learn of the happiness and success of students who attended her/his lectures. Probably, Natanson would have devoted himself to preparing his

lectures and to communicating with his students under such a belief, as speculated by many essays about him. One example of his effect on students can be seen in an essay written by Leopold Infeld (1958).

Infeld was a famous Polish physicist and coauthor with Einstein of *The Evolution of Physics*,[30] which is a very popular introduction to physics published in 1938. He was a rare person who was active in physics and in the international pacifist movement, together with Einstein, after the tragedies of Hiroshima and Nagasaki.

After Infeld returned to his mother country from Canada after World War II, he wrote an essay. In it, he had mentioned greatly impressed he was by Natanson's lectures at the university and had obtained his doctorate under Natanson's guidance. Although Infeld wished to continue studying physics as a researcher, he could not find an academic post at the university due to Poland's economic difficulties at the time. Subsequently, he had to work outside Kraków as a high-school teacher while waiting for a chance to return to academic research.

To encourage Infeld, Natanson sometimes sent letters with his papers attached; among these attached papers was the one on the statistical theory of radiation. However, the greatest desire of Infeld was to find his way back into academic life and he counted on Natanson to find a post for him. Unfortunately, this proved impossible and Infeld's extreme disappointment seems to have gradually led to hostility towards Natanson. Natanson's sympathy under these circumstances may have simply provoked Infeld's antipathy. Infeld eventually gained a post at Lvov with another person's help, and this opened the door for later collaboration with Einstein.

In Infeld's essay, he wrote:

> Dzisiaj dopiero oceniam lepiej skomplikowany charakter mego profesora. Widzę w nim człowieka niezdolnego do intryg, rycerskiego i szlachetnego. Człowieka wychowanego w dobrobycie, który obawia się kontaktu z życiem i jego brutalnością i bezwzględnością. Człowieka samotnego tak

[30] Jun Ishiwara translated it into Japanese. This translation has many versions today. If Infeld had mentioned Natanson in this work, Natanson's paper of 1911 (1911a or 1911b) could have been recognized by many Japanese people, including leading physicists.

> w nauce, jak i w życiu, dla którego bezosobowość w stosunkach z ludźmi była pancerzem ochronnym; takim pancerzem była jego niesłychana grzeczność posunięta do upokarzającej przesady. Naukowo był blisko, bardzo blisko, wielkich odkryć, np. sformułowania statystyki Bosego (Infeld 1958, p. 136).

It can be translated into English as:

> Only today can I assess better the complicated character of my professor. I can see in him a man incapable of intrigues, who is chivalrous and noble; a man raised in prosperity who is afraid of contact with life and its brutality and ruthlessness; a lonely man, both in science and in life, for whom impersonality in human relations was a protective armor; such armor was his remarkable politeness to a degree of humiliating exaggeration. He was scientifically close, very close to the great discoveries, e.g. the formulation of Bose statistics.[31]

His essay had been written ten years before he passed away. Putting aside the possibility that this might have been an excuse reflecting his official position in Poland at that time, his reminiscence can be taken at face value. However, I cannot find any evidence that during this period he reassessed Natanson's work with respect to that of the world's leading scientists during the same time. Nevertheless, Natanson would have certainly been happy to know that Infeld had finally recognized his unique contribution to the development of quantum statistical physics.

9. Remaining questions

Natanson passed away on February 26, 1937: he was 73 years old. This was thirteen years after publication of Bose's paper in 1924. In 1911, when Natanson published the paper in question, Heike Kamerlingh Onnes of Leiden discovered the superconductivity of metals. A letter dated December 17, 1927, one year after Onnes's death, was sent to Natanson from

[31] The translation of Polish text into English was provided by Professor Michal Kokowski from the Institute for the History of Science of the Polish Academy of Sciences.

Mieczysław Wolfke (Archive 2, p. 281). In this letter, he informed Natanson that he, together with Willem H. Keesom, had discovered a new phase of liquid helium 4 (He^4) at 2.3 kelvin when he was at Leiden. Their discovery was prior to Pyotr L. Kapitsa's discovery of the super fluidity of He^4 in 1937. These unusual phenomena are known today to be manifestations of macroscopic quantum effects of bosonic systems.

I presumed that Natanson must have known not only of Bose's paper of 1924 and Einstein's prediction of quantum condensation, today called Bose-Einstein Condensation (BEC), but also of the phenomena of liquid He^4. They should have been intriguing phenomena for theorists like Natanson. However, I have not been able to find comments about them so far from Natanson.

Why was Natanson so silent? I cannot accept the interpretation that Natanson had already lost his interest in quantum statistics as a theoretical physicist; it is more plausible that an unusual atmosphere or circumstances curtailed his academic activity during the years. BEC was verified to occur in cooled Rb^{87} gas by a group of scientists from Joint Institute for Laboratory Astrophysics (JILA) and National Institute of Standards and Technology (NIST) in 1995, 84 years after Natanson's paper (Anderson, 1995). This was the first terrestrial realization of BEC in a real gas system. The approximate bosonic character of composite fermions was theoretically discussed by Ehrenfest and Julius R. Oppenheimer in 1931 (Eherenfest, Oppenheimer 1931). This paper was accepted on December 23, 1930, six years before Natanson's death. Did Natanson know this paper?

10. Conclusion

This study[32] has provided us with many aspects on why the importance of Natanson's paper of 1911 was not recognized.

Victor Frederik Weisskopf wrote:

> There is a strong trend towards a clear- cut, universally valid answers that exclude different approaches. Whenever one way of thinking is developed with great force and

[32] I became interested in this topic just after coming back from *The International Workshop on Bose-Einstein Condensation (BEC93)*, held in Levico Terme in northern Italy, 31 May 31 to June 4, 1993, and have studied it on and off since then.

> success, other ways are unduly neglected. It was aptly expressed by Marcus Frierz, the Swiss physicist-philosopher: "The scientific insights of our age shed such glaring light on certain aspects of human experience that they leave the rest in even greater darkness" (Weisskopf 1981, pp. 22–23).

While Natanson's work has slipped into the darkness, his case does not seem to be exceptional because the contrast of the darkness is inherited unconsciously and emphasized by the routine way authors treat citations. I was also impressed by a passage in Stefan L. Wolff's paper:

> Citation is not a working technique, but also an ethics, the acknowledgement of obligations and a respect for truth (Wolff 2003).

As a future prospect, Natanson's fate could be avoided by the introduction of a digital tag in which all information on relevant papers published so far would be automatically accumulated and updated. This tag would be attached, being independent of authors, to the respective accepted article. Such a system would be very useful for readers of academic articles.

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