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The divergent histories of Bose-Einstein statistics and the forgotten achievements of Władysław Natanson (1864–1937)






Abstract

This article investigates the forgotten achievements of Władysław Natanson (1864–1937) related to the creation of Bose-Einstein statistics.

The introductory part of the article presents considerations regarding the methodology of history and the history of exact sciences, and then the divergent research perspectives that can be taken in the description of the history of Bose-Einstein statistics, as well as the author's integrated approach to this issue, which eliminates the disadvantages of these divergent views.

This integrated approach is then used to describe the achievements of Władysław Natanson related to the creation of Bose-Einstein statistics.

These achievements are presented against the background and in the context of discussions which – relatively sporadically –

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took place among various groups of researchers: historians and philosophers of science, physicists, sociologists of scientific knowledge in the 20th and 21st centuries.

These discussions have now been reordered here. They are followed by a presentation of the complete list of Natanson's publications regarding the subject. Also shown is his strategy to quote reliably the bibliography with regard to the explanation of the distribution of blackbody radiation and related issues.

Additionally, a list of scientists who knew Natanson's publications has been supplemented in the article and the precursorship of Natanson's achievements is explained. This is followed by a rebuttal of many erroneous or simplified statements about him and his achievements.

The already well-known terminological conventions have been recalled: "Bose statistics" and "Bose-Einstein statistics", as well as recently introduced: "Planck-Bose statistics" (1984), "Natanson's statistics" (1997)", "Natanson-Bose-Einstein statistics" (2005), "Planck-Natanson-Bose-Einstein statistics" (2011), and "Natanson statistics" (2013).

New terminological conventions have been introduced: "Boltzmann-Planck-Natanson statistics" and "Boltzmann-Planck-Natanson-Bose-Einstein statistics".

A side effect of this research is a discovery that Robert K. Merton – the author of the label 'Matthew effect' – chose the name of the effect using erroneous premises and the effect should therefore be named after its actual discoverer.

The article is accompanied by four appendixes: the first presents reflections on the methodology of historiography and historiography of exact sciences, the second – a commentary on the use of the terms: "Bose statistics", "Bose-Einstein statistics", "Einstein-Bose statistics" and "Planck-Bose statistics", the third – a very important letter by Max Planck to Władysław Natanson (of 25 January 1913), and the fourth – the excerpts of two letters from Sommerfeld to Rubinowicz (of 1 October 1919 and 1 November 1919).

Keywords: *Władysław Natanson, Ladislav Natanson, distribution of blackbody radiation, Bose statistics, Bose-Einstein statistics, Planck-Bose statistics, Natanson statistics, Natanson-Bose-Einstein statistics, Planck-Natanson-Bose-Einstein statistics, Boltzmann-Planck-Natanson statistics, Boltzmann-Planck-Natanson-Bose-Einstein statistics, divergent histories, integrated approach, precursorship, Matthew effect, R.K. Merton effect, methodology of history, methodology of the history of exact sciences*

Rozbieżne historie statystyki Bosego-Einsteina i zapomniane osiągnięcia Władysława Natansona (1864–1937)

Abstrakt

Artykuł bada zapomniane osiągnięcia Władysława Natansona (1864–1937) związane z powstaniem statystyki Bosego-Einsteina.

W części wstępnej artykułu wskazano rozbieżne perspektywy badawcze, jakie przyjmowano w opisie historii statystyki Bosego-Einsteina, a także autorskie zintegrowane ujęcie tego zagadnienia, które eliminuje wady tych rozbieżnych perspektyw.

Wspomniane zintegrowane ujęcie zastosowano następnie do opisania osiągnięć Władysława Natansona (1864–1937), związanych z powstaniem statystyki Bosego-Einsteina.

Dokonania Natansona przedstawiono na tle i w kontekście dyskusji, jakie toczyły się (stosunkowo sporadycznie) wśród różnych grup badaczy: historyków i filozofów nauki, fizyków, socjologów wiedzy naukowej w XX i XXI w.

Dyskusje uporządkowano oraz przedstawiono kompletną listę publikacji Natansona dotyczących omawianego zagadnienia. Wskazano także strategię rzetelnego cytowania przez Natansona bibliografii dotyczącej wyjaśnienia rozkładu promieniowania ciała doskonale czarnego i pokrewnych zagadnień; uzupełniono listę naukowców, którzy znali publikacje Natansona; skorygowano wiele błędnych lub uproszczonych stwierdzeń na temat Natansona i znaczenia jego publikacji, wyjaśniono kwestię prekursorstwa jego osiągnięć etc.

Przypomniano już znane konwencje terminologiczne: „statystyka Bosego” i „statystyka Bosego-Einsteina”, jak również niedawno wprowadzone: „statystyka Plancka-Bosego” (1984), „statystyka Natansona” (1997, 2013), „statystyka Natansona-Bosego-Einsteina” (2005) oraz „statystyka Plancka-Natansona-Bosego-Einsteina” (2011).

Wprowadzono nowe konwencje terminologiczne: „statystyka Boltzmann-Plancka-Natansona” i „statystyka Boltzmann-Plancka-Natansona-Bosego-Einsteina”.

Skutkiem pobocznym tych badań jest odkrycie, iż socjolog Robert K. Merton – autor określenia „efekt św. Mateusza” –

wybrał tę nazwę, posługując się błędnymi przesłankami i dlatego należy nazywać ten efekt nazwiskiem jego faktycznego odkrywcy.

Do artykułu dołączone są cztery dodatki: pierwszy przedstawia rozważania z zakresu metodologii historii i historii nauk ścisłych, drugi – komentarz dotyczący użycia terminów: „statystyka Bosego”, „statystyka Bosego-Einsteina”, „statystyka Einsteina-Bosego” oraz „statystyka Plancka-Bosego, trzeci – bardzo ważny list Maxa Plancka do Władysława Natansona z 25 stycznia 1913 r., a czwarty – fragmenty dwóch listów Sommerfelda do Rubinowicza z 1 października 1919 i 1 listopada 1919 r.

Słowa kluczowe: *Władysław Natanson, rozkład promieniowania ciała doskonale czarnego, statystyka Bosego, statystyka Bosego-Einsteina, statystyka Einsteina-Bosego, statystyka Plancka-Bosego, statystyka Natansona, statystyka Natansona-Bosego-Einsteina, statystyka Plancka-Natansona-Bosego-Einsteina, statystyka Boltzmann-Plancka-Natansona, statystyka Boltzmann-Plancka-Natansona-Bosego-Einsteina, rozbieżne historie, zintegrowane podejście, prekursorstwo, efekt św. Mateusza, efekt R.K. Mertona, metodologia historii, metodologia historii nauk ścisłych*

1. The divergent perspectives in studying the history of Bose-Einstein statistics and a postulate for an integration of research¹

To structuralize better our considerations on the history of Bose-Einstein statistics and Natanson's contribution to it, it is important to formulate here some remarks of a general nature.

Firstly, the scholars who researched the so-called Bose-Einstein statistics dealt with problems in physics and applied mathematics (statistics).

¹ The subject-matter of this article was analyzed by the author in several previous works: Kokowski 2009 (in Polish, only on p. 92, and fn. 3); 2011a and 2011b (I gave a lecture in English during a Prague conference and then a summary of the lecture and a presentation were published in the proceeding of the conference, but only in the CD-ROM version; these works were not reviewed, and did not receive DOI numbers); 2015 (I gave a lecture in English, but the lecture was not published in print or online). In consequence, one cannot find these works in print or online, and my views on this subject are not known to specialists. To remedy this, I present this article, which not only systematizes, but also greatly expands my previous analyses.

Secondly, these scholars functioned in certain scientific communities and thought collectives.² Their achievements are dependent on their talents and participation in scholar traditions or thought styles, because nobody can achieve success in science if they do not *stand on the shoulders of giants*.³

Thirdly, the creativity of scientists is measured by the quality of their publications, but an evaluation of these publications is not an easy matter. We can assume safely that the historical approach can be useful in this task. However, we cannot rule out *a priori* that the tools of scientometrics (which is a measurement of the development of science using mathematical tools, including a measurement of impact of scientific publications by a simple citation counting) can be useful too.

Fourthly, while our analyses of the context of justification must play an important part in our consideration, we must not neglect the importance of the context of discovery, since both these contexts interlace with each other in everyday practice of scientists.

Fifthly, between the justification and the discovery there is a whole intermediate field to persuade recipients (other scientists as well as a broader audience), and to mediate between different scientific camps; as a result we cannot neglect the rhetorical aspect of considerations, interactions between thought collectives, different interests of scientific camps including their political views, etc.

Sixthly, the history of Bose-Einstein statistics can be interpreted from divergent points of view that originate from such diverse disciplines as, for example: the teaching of physics, the history and philosophy of physics (and, generally, of the exact sciences), the sociology of scientific knowledge, the psychology of scientific discovery and scientometrics.

I suggest that these primary divergent points of view be treated as complementary perspectives of an integrated approach. And, from my point of view, the only reasonable approach to study the problem is to assume a certain thematic hierarchy of these points of view (in other words, these points of view are not important in the same sense). Firstly, we must carry on an extensive, detailed internal analysis of the development of scientific ideas (including the so-called philosophy in science).

² Fleck 1935/1979.

³ R.K. Merton 1965 (2nd ed. 1985; 3rd ed. 1993); *Wikipedia* [2019f](#); Kokowski [2012](#), pp. 57–58.

Then, we can look for the additional so-called ‘external’ explanations (originating from the philosophy of science, the sociology of scientific knowledge, the scientific rhetoric, political views, psychology, general philosophy, etc.).⁴

Seventhly and finally, when we want to write about the history of Bose-Einstein statistics, we should notice the two main approaches applied by researchers: a) the canonical approach, which is based on its “history” as seen by the authors of textbooks on statistical physics, and b) the historical approach. The latter gives us possibility of studying both, the so-called internal history of science, and the external one. We can thus study the internal history of physics (linked with the internal philosophy of physics), and the external history of physics. Thereby, the external history of physics is open to questions stemming from historical contexts defined by politics and philosophy, the sociology of scientific knowledge, the scientific rhetoric, the psychology of scientific discovery, as well as scientometrics. I am an advocate of the integrated approach linking in a hierarchical way both the internal and external factors.⁵

2. The different approaches in studying the history of Bose-Einstein statistics and Natanson’s achievements

2.1. The canonical approach, the university textbooks and Natanson

In the canonical interpretation of Bose-Einstein statistics there is no problem pointing to the real discoverers: they were simply Satyendra Nath Bose (1894–1974) and Albert Einstein (1879–1955), and nobody else. It is sufficient to look at the table below and compare only two formulas describing the distribution of particles over energy states for the

⁴ This integrated approach stems from the progress of history of science and philosophy of science, and sociology of scientific knowledge in the 20th century. It transcends the opposition of the ideas of “internal history of science” and “external history of science”.

⁵ Regarding the methodology of historiography and historiography of science assumed by the article author, see Appendix 1.

two statistics, to quote the articles of S.N. Bose 1924a (reprinted 2009a; English transl. 2009b); 1924b (reprinted 2009c; English transl. 2009d) and A. Einstein 1924 (reprinted 2015a, Doc. 283; English transl. 2015b, Doc. 283); 1925a (reprinted 2015a, Doc. 385; English transl. 2015b, Doc. 385); 1925b (reprinted 2015a, Doc. 427; English transl. 2015b, Doc. 427), as well as to show the textbooks, e.g. L.D. Landau, E.M. Lifshitz (1937–1939; ed. 1975, §54, pp. 180–181) / (English transl. 1958, §54, pp. 153–154); F. Hund (1956, § 92); K. Huang (1963, chapter 12); R.P. Feynman (1972, chapter 1.9) or *Wikipedia* (2019b).⁶

Table 1. The distribution of particles over energy states

Bose statistics (1924)	Bose-Einstein statistics (1924–1925)
$\frac{n_i}{g_i} = \frac{1}{\exp\left(\frac{\varepsilon_i}{k_B T}\right) - 1}$	$\frac{n_i}{g_i} = \frac{1}{\exp\left(\frac{\varepsilon_i - \mu}{k_B T}\right) - 1}$
where: n_i – population number of (indistinguishable) particles with energy ε_i , g_i – number of (distinguishable) sub-levels, n_i/g_i – probability of occupation level i with energy ε_i , μ – chemical potential. ⁷	

2.2. The internal history (and philosophy) of physics⁷

Changing the research perspective by using other “glasses” (i.e. other interpretative tools), which are sensitive to a detailed historical research, brings out new epistemic results. Thanks to the thoroughgoing

⁶ For a genesis and description of Bose’s and Einstein’s works, see Chapter 14 “Satyendra Nath Bose, Bose-Einstein-Statistics, and the Quantum Theory of an Ideal Gas” in: Mehra 2001, pp. 501–545.

⁷ In the canonical approach we ignore historical details and talk about “particles” irrespective of the fact whether we consider imponderable matter (quanta of light) or ponderable matter.

It is noteworthy that the formulas mentioned in the Table 1 are linked by a correspondence principle: for the limiting case, when the correspondence parameter “ $\mu/k_B T$ ” tends to zero, numeric values (predictions) of the second formula goes to the

works by, among others, M. Jammer (1966), J. Mehra, H. Rechenberg (1982–2000), S. Bergia (1987), A. Bach (1988; 1990), S. Varró (2006a, pp. 1–34; 2006b; 2007), we know that the history of the so-called Bose-Einstein statistics is very complicated and many scientists played important roles in it.

We can distinguish three subsequent main stages in this history: the first (preparatory) stage from the formulation of the laws of electrodynamics and the principles and laws of statistical physics but before formulation of the black-body radiation law; the second stage, the formulation of the black-body radiation law, and the third stage, the explanation of the black-body radiation law and the formulation of the so-called Bose statistics and then of its generalization Bose-Einstein statistics.⁸

In the first (preparatory) stage the laws of electrodynamics and the principles and laws of statistical physics were formulated, including the entropy-probability relationship found by Boltzmann (see Bach 1988; 1990, p. 2), the Maxwell-Boltzmann distribution, and the Wien-Jeans law of radiation.

In the second stage Max Planck (1900a) discovered the black-body radiation law. The law is described by the following formulas:

Table 2. The black-body radiation law

$$\rho_\nu(\nu, T) = 8\pi\nu^2 c^{-3} \frac{h\nu}{\exp\left(\frac{h\nu}{kT}\right) - 1},$$

numeric values (predictions) of the first formula. This is not an accidental feature and no anachronism. It is a manifestation of applying the hypothetico-deductive method of correspondence-oriented thinking by researchers of the so-called exact sciences (see Kokowski 1996; 2001; 2004; 2006; 2015c). Therefore: a) I do not agree with Jean Bricmon (2015), who – going on footnotes of Thomas S. Kuhn, Paul Feyerabend, and the sociology of scientific knowledge – declares that there is no scientific method and it is not a problem, and b) I do agree with Elliott Sober (2015), who is sure that the scientific method is not a myth and there are general normative principles that govern every science.

⁸ Of course, this three-part division is only a conventional division.

$$\rho_\nu(\nu, T) = 8\pi\nu^2 c^{-3} U(\nu, T),$$

$$U(\nu, T) = \frac{h\nu}{\exp\left(\frac{h\nu}{kT}\right) - 1}$$

where:

$\rho_\nu(\nu, T)$ – the spectral energy density of radiation in cavity in thermal equilibrium at absolute temperature T per unit volume and per frequency unit;

$8\pi\nu^2 c^3 d\nu$ – the number of modes of oscillation (states) in the frequency interval $[\nu, \nu + d\nu]$ per unit volume;

$U(\nu, T)$ – the mean energy of oscillator of frequency ν and absolute temperature T .

In the third stage, the subsequent theoretical derivations and justifications of the black-body radiation law by Max Planck (1900b; ...) were criticized by other scientists: Joseph Larmor, Charles Thomson Rees Wilson, Peter Debye, Hendrik Lorentz, Władysław Natanson, Abram Fyodorovich Joffé, Paul Ehrenfest, Jun Ishiwara, Iurii Aleksandrovich Krutkov, Mieczysław Wolfke, Heike Kamerlingh Onnes, Maurice de Broglie, Arthur H. Compton, Wolfgang Pauli, Viktor R. Bursian, Otto Halpern, and finally Satyendra Nath Bose in 1924, and Alfred Einstein in 1924–1925.⁹ Regarding Bose's and Einstein's contributions: Bose in-

⁹ This third stage was analysed by dozens of scholars, mainly physicists and historians of physics. There are two groups of such scholars. The first group, which overlooked Natanson's achievements (a majority of physicists and historians of physics); and the second group, which noticed his achievements.

To the first group belong, among others, B.I. Spasskiĭ (1964, chap. 19, §73); L.D. Landau, E.M. Lifshitz (3rd ed. 1976; Engl. transl. 1986, §37, 54, 55); F. Hund (1956, § 92); K. Huang (1963, chapter 12); H. Kangro (1970/1976); R. Feynman (1972, chapter 1.9); T.S. Kuhn (1978); M. Toda, R. Kubo, N. Saitō (1978; Engl. transl. 1983, chapter 3.1.3); A. Pais (1979, paragraph VI; 1982, repr. 2005, chapter 23); C. Domb (1995); C.A. Gearhart (2002); H. Kragh (2002); R. Fitzpatrick (2006); A. Michelangeli (2007); D. Monaldi (2009); E.P. Canals, T. Sauer (2010a); *Wikipedia* (2019b; 2019d).

To the second group belong, among others, E.T. Whittaker (1953); F. Hund (1967); A. Hermann (1969; Engl. transl. 1971); A. Kastler (1981); B. Średniawa (1985;

roduced statistics for radiation, called now Bose statistics, and Einstein, generalizing Bose's approach, introduced statistics both for imponderable matter (radiation) and of ponderable matter (material vibrators / atoms), called now Bose-Einstein statistics.¹⁰

However, the understanding of this entire three-part story, including the reasons why all of the authors criticized Planck's approaches, is not the aim of this article.¹¹ I restrict below only to illuminate the issue of the reception of Natanson's views in his times and later.

3. Natanson's achievements in focus

In this section, in order to systematize the knowledge on Natanson's achievements dispersed among different kinds of specialists, I will try to summarise the discussions about the issue held among specialists and add my own comments. To achieve this aim I will try to answer key questions regarding this issue.

3.1. Elementary issue: How many works did he write on the subject?

It is an elementary issue for a positivistic methodology of the history of science to establish the number of works that Natanson wrote about the statistics of imponderable matter (black-body radiation) and of ponderable matter (material vibrators).

As far as I know, he wrote the following list of works on the subject mentioned.

[1997](#); 2000; 2001; [2007](#); A. Pais (1986); S. Bergia (1987); A. Bach (1988; 1990); O. Darrigol (1988; 1993); B. Lange ([1992a](#); [1992b](#); 1997a; [1997b](#)); L.J. Boya ([2003](#)); J. Spalek ([2005](#); [2006](#); 2009); S. Varró ([2006a](#); [2006b](#)); A. Borrelli ([2009](#)), "R. Minamida" (N. Nagasawa) (2009a); M. Kokowski (2009; 2011a; 2011b); M. Waniek, K. Hentschel ([2011](#)); B.R. Masters (2013); N. Nagasawa ([2018](#)); K. Hentschel ([2018](#)) – this group is not homogeneous: its representatives declare a whole spectrum of views (I will explain it later).

¹⁰ About these terms see Appendix 2.

¹¹ This matter is complicated and worthy a separate detailed book. For a general introduction to the history of physical ideas and the problem-situation see Mehra, Rechenberg [2001](#), pp. 557–578. One of the important threads of this history is to explain a *combinatorial and physical problem*: the distribution of indistinguishable particles over energy states.

- 1) O teorii statystycznej promieniowania. On the Statistical Theory of Radiation (presented: 6 March 1911; published: circa 10 April 1911). *Bulletin International de l'Académie des Sciences de Cracovie, Classe des Sciences mathématiques et naturelles. Série A: Sciences mathématiques. Anzeiger der Akademie der Wissenschaften in Krakau. Mathematisch-Naturwissenschaftliche Klasse. Reihe A: Mathematische Wissenschaften*, pp. 134–148. (in English) & offprint. Hereafter: Natanson 1911a.

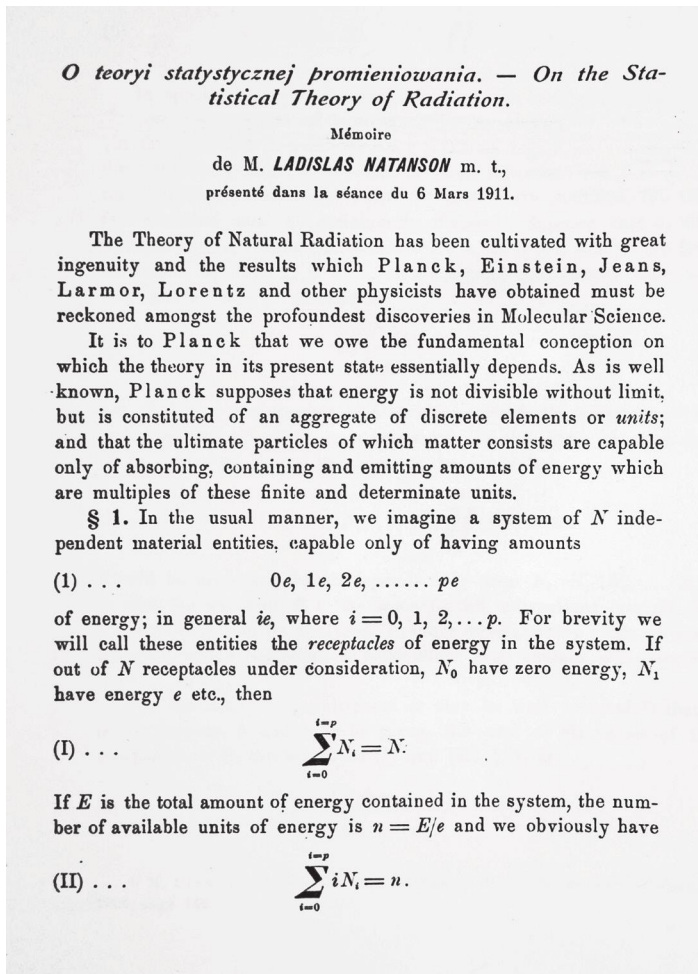


Fig. 1. The front page of “On the Statistical Theory of Radiation” (presented: 6 March 1911; published: circa 10 April 1911). Source: © Biblioteka Jagiellońska; photo: © Michał Kokowski.

- 2) O promieniowaniu (On Radiation) (1st vers.; 19 July 1911). [In:] *Księga pamiątkowa XI Zjazdu Lekarzy i Przyrodników Polskich w Krakowie, 18–22 lipca 1911* {*Proceedings of the 11th Congress of Polish physicians and natural scientists in Krakow, 18–22 July 1911*} (Kraków: Komitet Gospodarczy, 1911), pp. 144–160. Available online: <https://jbc.bj.uj.edu.pl/dlibra/doccontent?id=278801>. Hereafter: Natanson 1911b).

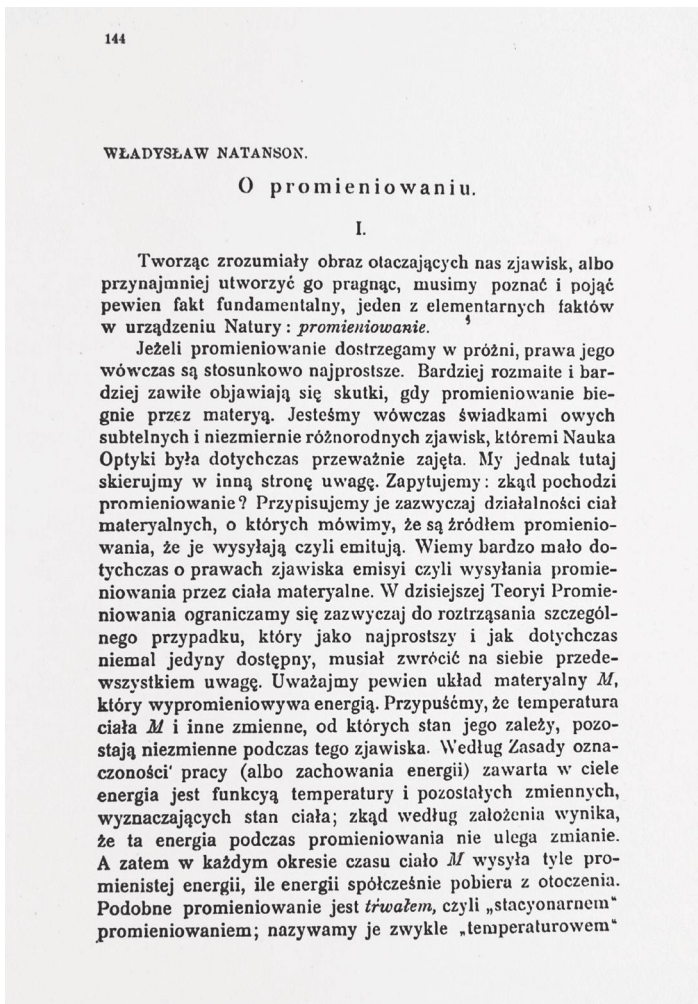


Fig. 2. The front page of “O promieniowaniu” (1st version, 19 July 1911). Source: © Biblioteka Jagiellońska; photo: © Michał Kokowski.

- 3) Über die statistische Theorie der Strahlung (received: 29 April 1911; published: 15 August 1911). *Physikalische Zeitschrift* 12, pp. 659–666 {it is a translation of Natanson’s first paper (1911a)}; & offprint (in German). The translation was made by Max Iklé, when the chief editor of the journal was Friedrich Krüger. Hereafter: Natanson 1911c.

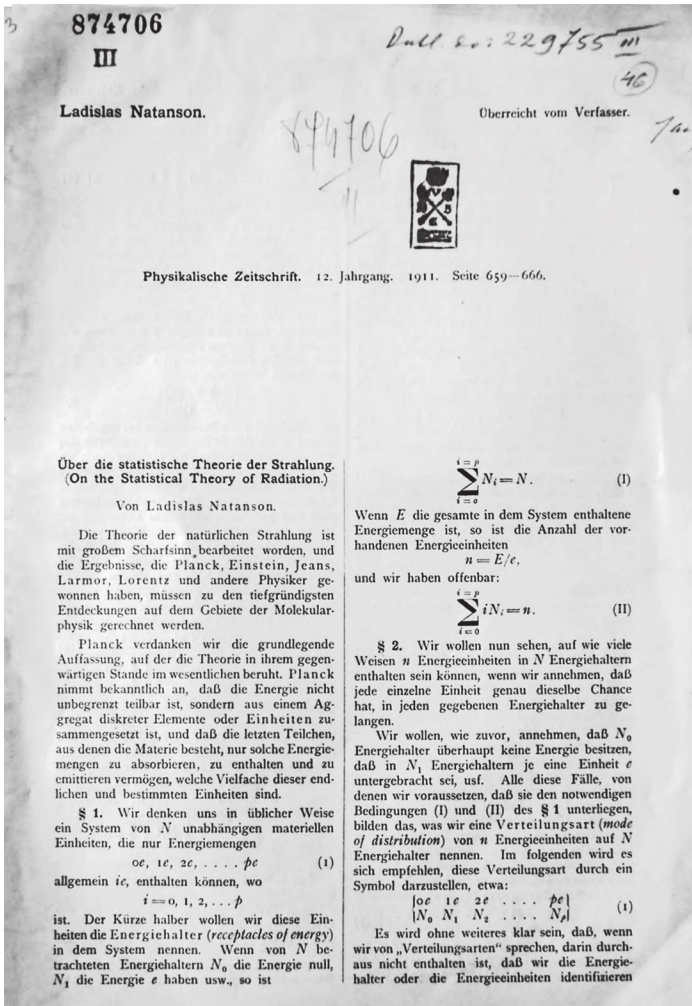


Fig. 3. The front page of “Über die statistische Theorie der Strahlung” (received: 29 April 1911; published: 15 August 1911). Source: © Biblioteka Jagiellońska; photo: © Michał Kokowski.

- 4) On Radiation (1st vers.) – Offprint 1912 of Natanson 1911b (in Polish). Hereafter: Natanson 1912a.

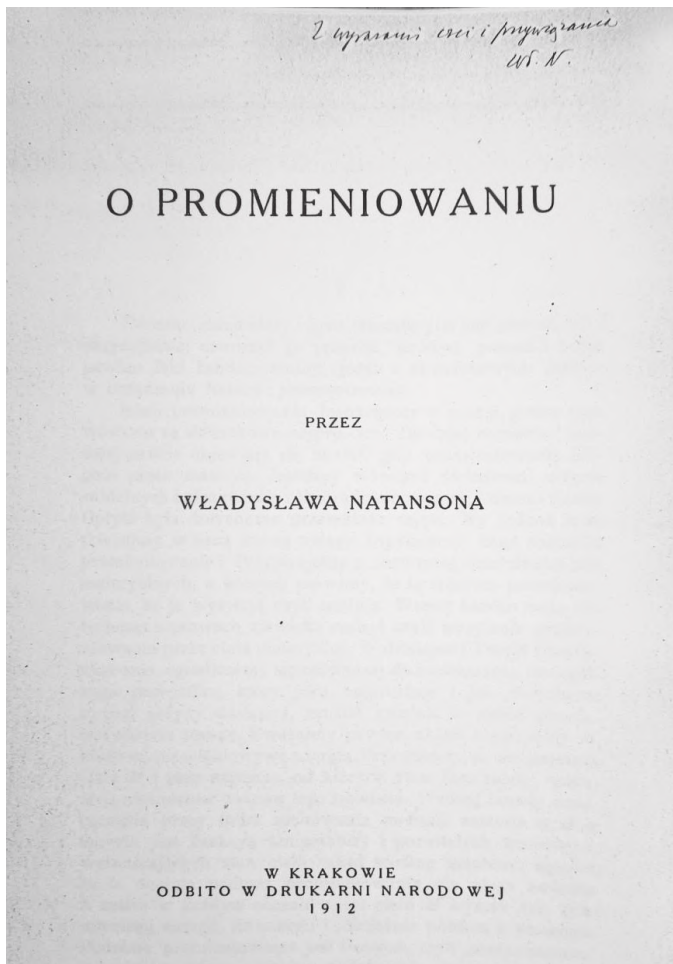


Fig. 4. The front page of “On Radiation” (1st version) – offprint 1912 of Natanson 1911b. Source: © Biblioteka Jagiellońska; photo: © Michał Kokowski.

- 5) O zawartości energii w ciałach materyalnych – On the Energy-content of material bodies (presented on 8 January 1912; published: April 1912). *Bulletin International de l'Académie des Sciences de Cracovie, Classe des Sciences mathématiques et naturelles. Série A: Sciences mathématiques. Anzeiger der Akademie der Wissenschaften*

in Krakau. *Mathematisch-Naturwissenschaftliche Klasse. Reihe A: Mathematische Wissenschaften*, pp. 95–102 & offprint (in English). Hereafter: Natanson 1912b.

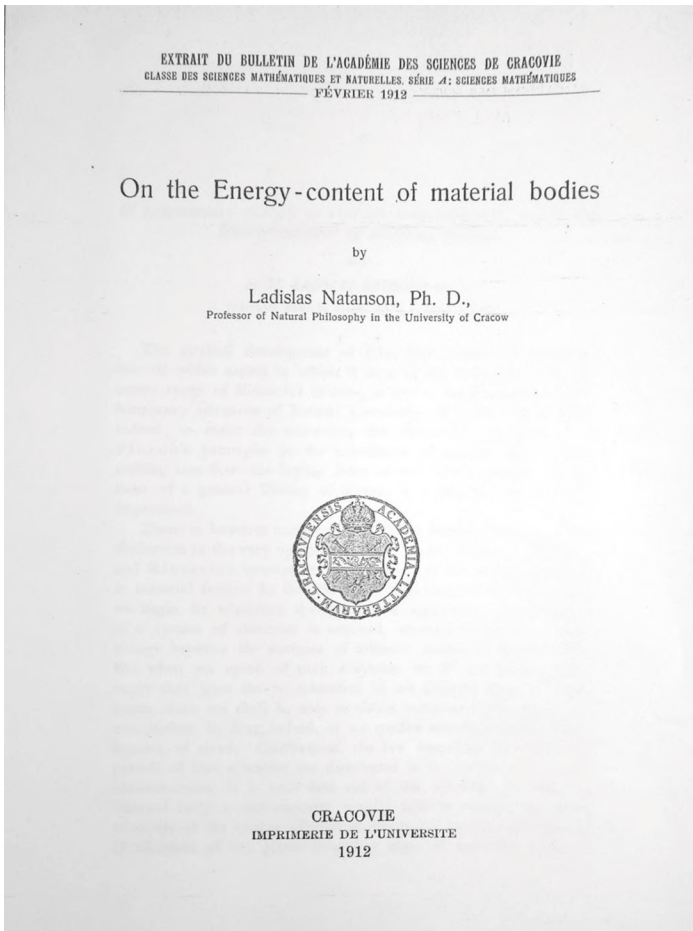


Fig. 5. The front page of “On the Energy-content of material bodies” (presented on 8 January 1912; published: April 1912). Source: © Biblioteka Jagiellońska; photo: © Michał Kokowski.

- 6) *Zasady Teorii Promieniowania (Principes de la Théorie du Rayonnement)* (in Polish). *Prace Matematyczno-Fizyczne* 24, pp. 1–88. Warszawa: Wydawnictwo Redakcji Prac Matematyczno-Fizycznych. Available online (at “Polska Biblioteka Wirtualna Nauki”,

„Kolekcja Matematyczna”): <http://matwbn.icm.edu.pl/ksiazki/pmf/pmf24/pmf2411.pdf>. Hereafter: Natanson 1913. (*The article, though written only in Polish, is the most important Natanson’s work on the theory of radiation and related matters.*)

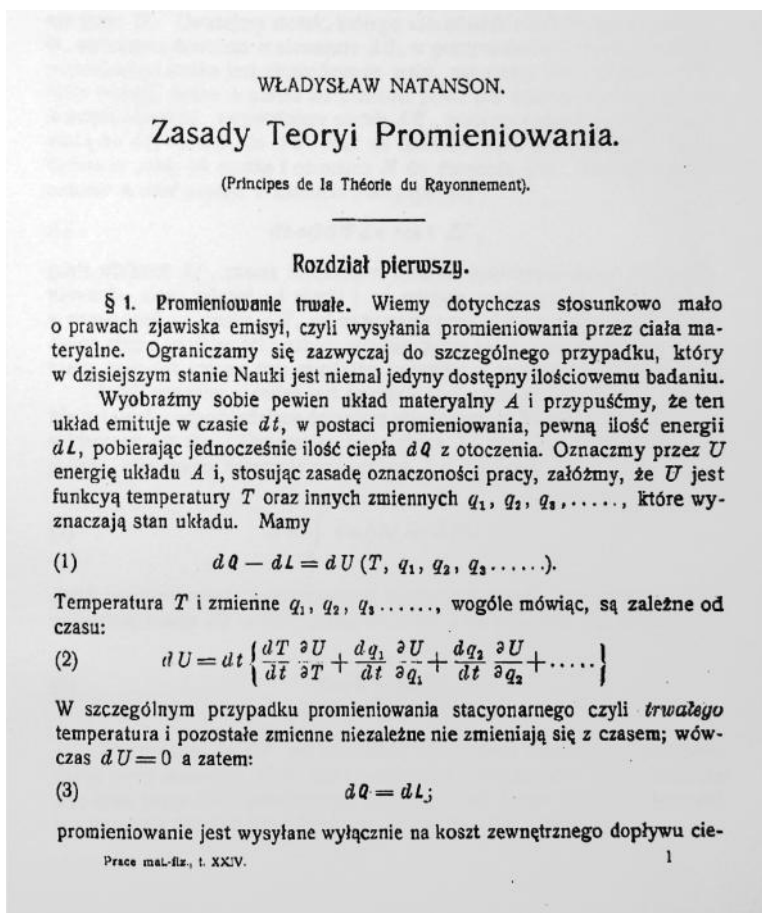


Fig. 6. The front page of “Zasady Teorii Promieniowania” (“Principes de la Théorie du Rayonnement”) (in Polish). Source: © Polska Biblioteka Wirtualna Nauki, Kolekcja Matematyczna; photo: © Michał Kokowski.

- 7) On Radiation (*2nd vers. with changes*) (1924). [In:] Natanson 1924a, pp. 125–151 (in Polish). Hereafter: Natanson 1924b; before the publication of Bose’s first paper.

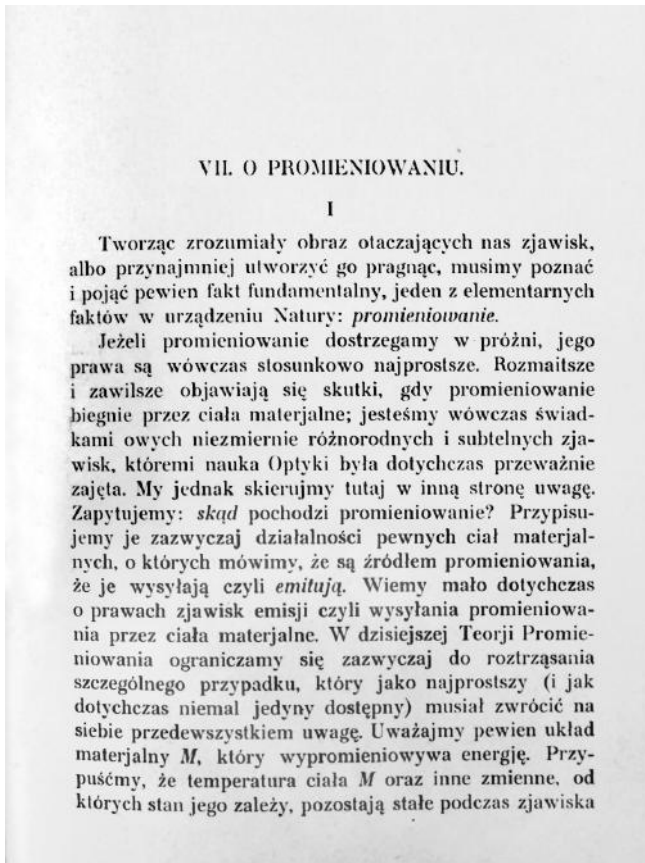


Fig. 7. The front page of “On Radiation” (2nd vers. with changes, 1924).

Source: © Biblioteka Jagiellońska; photo: © Michał Kokowski.

3.2. What works does Natanson mention in the bibliography of his works?

From the current research in the science of science, which includes bibliometrics, it is known that the authors of scientific papers have often a very serious problem with the reliability of quoting the publications they use during the preparation of their own publications.¹² Therefore, it is worth examining how Natanson dealt with this problem in his times.

¹² Kokowski [2015b](#).

In the article L. Natanson 1911a: On statistical theory of radiation. *Bulletin International de l'Académie des Sciences de Cracovie A. Sciences mathématiques. Anzeiger der Akademie der Wissenschaften in Krakau. Mathematisch-Naturwissenschaftliche Klasse. Reihe A: Mathematische Wissenschaften* (presented: 6 March 1911; published circa 10 April 1911), and its translation into German: Natanson, Ladislas (Władysław) 1911c: Über die statistische Theorie der Strahlung. *Physikalische Zeitschrift* (received: 29 April 1911; published: 15 August 1911). “Nach *Bulletin de l'Académie des Sciences de Cracovie* (A), pp. 134–148, 1911 (aus dem Englischen übersetzt von Max Iklé; eingegangen 29. April 1911)” Natanson mentions the following nine works:¹³

- Boltzmann, Ludwig 1872: Weitere Studien über das Wärmegleichgewicht unter Gasmolekülen. *Sitzungsberichte d. K. Akad. d. Wiss. zu Wien* II Abt, 66, pp. 275–370 (October 1872).
- Boltzmann, Ludwig 1877: Über die Beziehung zwischen dem zweiten Hauptsatze der mechanischen Wärmetheorie und der Wahrscheinlichkeitsrechnung resp. den Sätzen über das Wärmegleichgewicht. *Sitzungsberichte d. K. Akad. d. Wiss. zu Wien* II Abt, 76, pp. 373–435 (October 1877).
- Debye, Peter 1910: Der Wahrscheinlichkeitsbegriff in der Theorie der Strahlung. *Annalen der Physik* 33, pp. 1427–1434.
- Einstein, Albert 1907: Die Plancksche Theorie der Strahlung und die Theorie der spezifischen Wärme. *Annalen der Physik* 22, pp. 180–190. (Available online: https://www.physik.uni-augsburg.de/de/lehrstuehle/theo2/adp/history/einstein-papers/1907_22_180-190.pdf).
- Jeans, James H. 1910: On non-Newtonian Mechanical Systems, and Planck's Theory of Radiation. *Philosophical Magazine* 20, pp. 943–954 (p. 953). (Available online: <https://archive.org/details/londonedinburg6201910lond/page/n4>).
- Larmor, Joseph 1909: Bakerian Lecture. On the Statistical and Thermodynamical Relations of Radiant Energy. *Proceedings of Royal Society of London (A)* 83, pp. 82–85. (Available online: <https://www.jstor.org/stable/92866>).

¹³ He used a short version of the citation, accepted by physicists (i.e. without giving the publication title, without the scope of pages of the publication).

- Lorentz, Hendrik A. 1910: [Alte und neue Fragen der Physik]. *Physikalische Zeitschrift* 11, [pp. 1234–1257], p. 1253.
- Planck, Max 1906: *Vorlesungen über die Theorie der Wärmestrahlung* (1st ed.).
- Wilson, Harold A. 1910: On the Statistical Theory of Heat Radiation. *Philosophical Magazine* 20, pp. 121–125. (Available online: <https://archive.org/details/londonedinburg6201910lond/page/n4>).

In Natanson, Ladislas (Władysław) 1912b (presented on 8 January 1912; published: April 1912): O zawartości energii w ciałach materialnych – On the Energy-content of material bodies. *Bulletin International de l'Académie des Sciences de Cracovie A. Sciences mathématiques. Anzeiger der Akademie der Wissenschaften in Krakau. Mathematisch-Naturwissenschaftliche Klasse. Reihe A: Mathematische Wissenschaften*, pp. 95–102, the author mentions two theories:

- Planck's theory of radiation;
- Einstein's theory of specific heat;

and two works:

- Natanson, Ladislas (Władysław) 1911a: On statistical theory of radiation. *Bulletin International de l'Académie des Sciences de Cracovie A*, pp. 134–148;
- Duhem, Pierre 1911: *Traité d'Energétique ou de Thermodynamique Générale*.

In the work: Natanson, Ladislas (Władysław) 1913: Zasady teorii promieniowania (Principes de la Théorie du Rayonnement). *Prace Matematyczno-Fizyczne* 24, pp. 1–88. Warszawa: Wydawnictwo Redakcji Prac Matematyczno-Fizycznych, the author mentions all possible works on the subject written by authors in German, English, French, Italian and Polish – nearly 200 references, including for example P. Ehrenfest's papers of 1906; 1911a, and T&P. Ehrenfest of 1911.¹⁴

The article “O promieniowaniu” (On radiation), marked as Natanson 1911b, 1912a (i.e. offprint of 1911b) and 1924b (i.e. 1924a, pp. 125–151; it is a revised version of 1911b), is a review essay. It does not only regard radiation but also theories of gases, of liquids and solid states,

¹⁴ I will continue these considerations later in the article.

with quanta of energy as a key joining all these issues. The article was published without a bibliography, but the dates and the names of authors of main results are mentioned. In the case of the version of 1924, the name of Bose is still omitted, since Natanson wrote his article before Bose's articles (1924a;¹⁵ 1924b¹⁶).

3.3. Who was the first scientist to appreciate very highly the achievements of Natanson in quantum statistics?

The received answer for this question is clear. It was Friedrich Hund (1896–1997), a German physicist and historian of physics.¹⁷

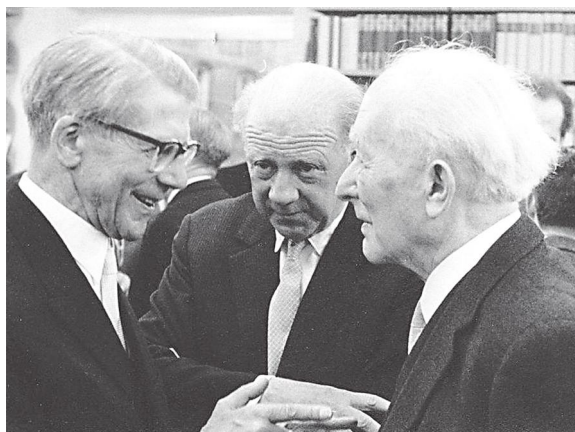


Fig. 8. Friedrich Hund, Werner Heisenberg and Max Born (Hund's seventieth birthday, Göttingen, 4 February 1966). Source: http://upload.wikimedia.org/wikipedia/commons/thumb/6/60/Hund_Heisenberg_Born_1966_Göttingen.jpg/800px-Hund_Heisenberg_Born_1966_Göttingen.jpg.

He stated so in his monograph of 1967: *Geschichte der Quantentheorie* (Mannheim: Bibliographisches Institut, 1967); English translation: *The history of quantum theory*. Transl. by Gordon Reece (London: Harrap, 1974); Italian translation by G. Longo: *Storia della teoria dei quanta* (Bologna: Bollati Boringhieri, 1975); Japan translation: by K. Yamazaki 1978 and Russian translation: *Istorija kvantovoj teorii* (Kiev: Naukova Dumka,

¹⁵ Reprinted [2009a](#), English transl. [2009b](#) / English transl. [1976](#).

¹⁶ Reprinted [2009c](#), English transl. [2009d](#).

¹⁷ Cf. Hund, Hentschel, Tobies [1996](#); *Wikipedia* [2019c](#).

1980). It happened only after 56 years from the appearance of Natanson's first work (Natanson 1911a).

But why did it happen so late? And is it true that only in 1967 did Hund – as a historian of physics – first appreciate Natanson's achievements in quantum statistics? Moreover, did Natanson's contemporary scientists value his work in the 1910s and 1920s? I will try to shed some light on these problems later in this article.

3.4. What is the essence and rank of Natanson's achievements?

Researchers commenting Natanson's achievements answered for this question in three complementary ways.

Firstly, Natanson's name was linked with the name of Satyendra Nath Bose and his statistics of *"light quanta"*. This strategy was applied by such scientists as L. Infeld (1958); F. Hund (1967/1974); A. Herman (1969, 1971); A. Kastler (1981); B. Średniawa (1985; [1997](#); 2000, pp. 454–455; 2001, pp. 105–107; [2007](#)); B. Lange ([1992a](#); [1992b](#); 1997a; [1997b](#)), and J. Spalek ([2005](#); [2006](#); 2009).¹⁸

According to Leopold Infeld (1958, p. 136; 1964b, pp. 35–36):

[Natanson] was close, remarkably close to the great scientific discoveries, such as the formulation of Bose statistics [translation — M.K.].¹⁹

¹⁸ S. Bergia (1987), A. Bach (1988; 1990), J.J. Stachel ([2000](#)), and M. Kokowski (2009; 2011a; 2011b) do not belong to this list. I will explain their stances below.

¹⁹ Of course, this side-note by Infeld, spoken within his reminiscences on his teacher, does not diminish the importance of the role of Friedrich Hund's priority (1967) in appreciating the meaning of Natanson's thought in the history of quantum physics (see above section 3.3).

Incidentally, in order to understand this Infeld's view better, it is worth sketching his attitude to his teacher Natanson. They were both Polish Jews, physicists and talented writers of popular books, however they differed considerably in social, philosophical and political terms. Infeld came from a poor and uneducated family, and Natanson had a rich and educated burgeois background. The former was an atheist and socialist, and the latter (Władysław *Szełiga* Natanson) in 1900 converted from Judaism to Catholicism (cf. Mises [1938a](#), pp. 144–150). In addition, Natanson did not help Infeld to get a job at university.

Unfortunately, Infeld – while writing his first famous autobiography (1941, reprint in 1980, 2006 & 2017) – repeatedly diverted from the truth. Among others, he

In contrast, according to Friedrich Hund (1967, pp. 25–26, 134, 153–154; (English transl.) 1974, pp. 30, 145; (Russian transl.) 1980, pp. 26, 123) *Natanson was the first who formulated Bose statistics of “light quanta”*:

This method of counting events, that Natanson made, is exactly the one, which Bose later made for light quanta, and is now called Bose statistics (Hund 1974, p. 30).²⁰

created an untrue, much exaggerated, picture of Polish anti-Semitism, and an entirely misguided picture of his teacher Natanson: “The only lecturer in mathematical physics [in Kraków] was an old, completely detached professor, delighted with the smoothness and external beauty of his lectures and not really giving a damn whether he inspired anyone or not. For thirty years he had lectured in Cracow and had never had a Ph.D. student” (Infeld 1941, repr. 2006, p. 88).

In fact, Natanson had five Ph.D. students: Stanisław Loria (1907), Waclaw Staszewski (1917), Leopold Infeld (1921), Józef Miczyński (1922), Stefan Szymon Rozental (1928), and eleven others students, who received a Ph.D., had been earlier peer-reviewed by him (cf. Dybiec 2009, pp. 30–38).

Natanson was also a very good teacher, which Infeld explained himself clearly in his later essay (1958, pp. 130–136), after his comeback to Poland. It appears that Infeld admired Natanson for his great intellectual culture, but he had held a grudge against him, because although he was – according to Infeld – his only scientific pupil (which was not true – see above): “he did not teach me the technique of scientific work and did not provide me with the right conditions to conduct such work” (“nie nauczył mnie techniki pracy naukowej i nie dal mi warunków do tej pracy” (Infeld 1958, p. 134). Therefore, it is not psychologically surprising that the pupil has not found the time to comment on the teacher’s achievements in more detail (cf. Natanson 1933/1958, pp. 115–119; Infeld 1958, pp. 130–136).

In his new autobiographical essays written in Polish (1954; 1964; 1967), Infeld tempered his views presented in his first autobiography (1941). Nevertheless, he repeated in 1967 his sentiments to his teacher: “So far, I do not understand why Professor Natanson did not offer me a job as his assistant. Certainly no one at the university would oppose the will of one of the most important professors. Of course I was a Jew. But was that enough of a reason? Perhaps, but today I think it’s not the only one. Professor Natanson must have been disgusted with the idea of having an assistant”. (“Dotychczas nie rozumiem, dlaczego profesor Natanson nie zaproponował mi asystentury. Na pewno nikt na uniwersytecie nie sprzeciwiłby się woli jednego z najważniejszych profesorów. Oczywiście byłem Żydem. Ale czy to powód wystarczający? Może tak, ale dzisiaj sądzę, że nie jedyny. Profesor Natanson musiał mieć wstręt do idei posiadania asystenta” (Infeld, 1967, p. 187)).

For explaining Infeld’s wrong opinions, especially the allegedly Polish radical anti-Semitism among the Polish prewar academia, cf. Wróblewski 2017, pp. 71–82. See also Hurwic 1968, reprinted 2016, pp. 405–417.

²⁰ “Die Abzählung, die Natanson verdeutlicht hat, ist genau die, die Bose später auf Lichtquanten anwandte und die man jetzt Bose-Statistik nennt” (Hund 1967, p. 26).

Bose statistics of light quanta was thus led to Planck radiation formula. This method of counting events for indistinguishable particles, which had already been perfectly recognized by Natanson in 1911, was subsequently to be called Bose statistics (Natanson's work of 1911, had of course been forgotten by 1924). It was not until some years later that the alternative possibility of the quantum statistics of the indistinguishable particle, that of Fermi statistics, was considered (Hund 1974, p. 145).

This thesis of F. Hund was accepted later by some researchers, such as A. Hermann (1971, p. 141), A. Kastler (1981), B. Średniawa (1985, pp. 89–90; [1997](#), pp. 14–16 (and repeated by K. Czapla [2005](#), p. 55); 2000, pp. 454–455; 2001, pp. 105–106; [2007](#), pp. 713–714), A. Bach (1990, pp. 1–2), L.J. Boya ([2003](#), p. 110), K. Hentschel (2006, p. 15; [2018](#), pp. 81–86), S. Varró (2006a, pdf version, pp. 14–16; 2006b, pdf version, p. 4; 2007, pp. 161–162), B. Lange ([1992a](#); [1992b](#); 1997a; [1997b](#)), and “Roh Minamida” (2009)/N. Nagasawa ([2018](#)).

For example, according to Armin Hermann (1971, p. 141):

Natanson (besides Max Planck, Albert Einstein and Paul Ehrenfest) was one of the first to establish a background for deeper understanding of the nature of quantum physics.²¹

According to Alfred Kastler (1981):

Natanson discovered Bose-Einstein statistics 13 years before Bose and Einstein.

According to Bronisław Średniawa (1985, p. 89):

In 1911 Natanson turned his scientific interest towards quantum theory. The papers *On the Statistical Theory of Radiation* [...], and its German version published in that year belong to the most important Natanson's publications. They contain the first formulation of quantum statistics, which was 13 years later rediscovered independently by Indian

²¹ The statement was later cited by, among others, Średniawa [1997](#), p. 16, and repeated by Czapla [2005](#), p. 56.

physicist Bose [...] and developed by Einstein and is today called “Bose-Einstein statistics” [my emphasis – M.K.].²²

According to Alexander Bach:

Natanson was the first who formulated Bose statistics of “light quanta”; the statistical assumptions of Bose are contained in a work of Natanson (1911c) (Bach 1990, pp. 1–2).

According to Bogdan Lange (1997a, p. 526):

The analysis and comparison I conducted show that procedures employed by Natanson and Bose are identical. Therefore, Hund (1974, p. 145) was right when he said, “The Bose statistics of light quanta was thus led to Planck radiation formula. This method of counting events for indistinguishable particles, which had already been perfectly recognized by Natanson in 1911, was subsequently to be called Bose statistics (Natanson’s work of 1911, had of course been forgotten by 1924) [...]”.

For this reason Bogdan Lange (1997a) uses the phrase “Natanson’s statistics”, and Peter Mittelstaedt (2013, pp. 83–85) “Natanson statistics” (however, he does not quote Lange’s article).

A separate opinion was expressed by a physicist Józef Spalek (2005; 2006; 2009)²³:

We should talk about Natanson-Bose-Einstein statistics, with the reservation that Natanson assigned indistinguishability to photons absorbed in “atoms of energy”, but Bose assigned statistical properties to the radiation itself in this sense that he considered the number of photon states as restricted only by geometry of container (it is the accepted view today) [summarised by M.K.].

A similar thesis was formulated by two historians of science: Magdalena Waniek and Klaus Hentschel (2011, p. 42):

²² Those statements were later repeated by Czapla 2005, pp. 55–56.

²³ Knowing B. Średniawa’s works, J. Spalek inherits some views of earlier scholars of Natanson’s achievements (particularly of F. Hund’s).

Actually, the quantum statistics named nowadays after Bose and Einstein would have to be called Planck-Natanson-Bose-Einstein statistics. This confirms the first law of the history of science – namely that (almost) no scientific result is named after the people who actually discovered it first [translated by M.K.].²⁴

Secondly, the researchers commenting Natanson's achievements thought that he was the first to understand the statistical foundations of Planck's law of black-body radiation. It was claimed by the following scholars: F. Hund (1967, pp. 26, 153–154), Kangro (1970/1976, p. 219, fn. 212), A. Kastler (1979; 1983), S. Bergia (1987, pp. 233–236; repr. 2009, pp. 343–346), T. You Wu (1986, p. 40), A. Bach (1990, p. 24), B. Średniawa (1985, pp. 89–90; 1997, pp. 14–16; 2000, pp. 454–455; 2001, pp. 105–106; 2007, pp. 713–714), A. Pais (1986, pp. 283, 294), A. Kojevnikov (2002, p. 198), J. Spalek (2005; 2006; 2009), S. Varó (2006a, pdf version, pp. 14–16), P. Enders (2007, p. 87), M. Waniek, K. Hentschel (2011, p. 42), B.R. Masters (2013, p. 43), O. Passon and J. Grebe-Ellis (2017, p. 7).

For example, according to Friedrich Hund (1967, pp. 153–154/1974, p. 167):

In the course of deriving his radiation formula (1900), Planck had applied a noteworthy form of statistics for the distribution of energy quanta among the oscillators: equally probable events were the occupation numbers of the oscillators. In 1911 Natanson recognized this as containing a possible form of the statistics of indistinguishable particles. Bose applied the same form of statistics (1924) to light particles, and Einstein – to gas molecules: he showed that the fluctuations in such a gas behaved as if they were caused by both particles and waves (Hund 1974, p. 167).²⁵

²⁴ “Eigentlich müsste die heute nach Bose und Einstein benannte Quantenstatistik somit Planck-Natanson-Bose-Einstein-Statistik heißen. Hier bestätigt sich auf's Neue der erste Hauptsatz der Wissenschaftsgeschichte – dass nämlich (fast) kein wissenschaftliches Resultat nach dem Namen seines tatsächlichen Erst-Entdeckers benannt ist” (Waniek, Hentschel 2011, p. 42).

²⁵ “Bei der Ableitung seiner Strahlungsformel (1900) verwandte Planck eine bemerkenswerte Statistik für die Verteilung von Energiequanten auf Oszillatoren:

According to Hans Kangro (1970/1976, p. 219, fn. 212):

Ladislav Natanson was the first to recognize the reason why Planck's statistics must in contrast differ from "classical" statistics (Natanson 1911[c], [pp.] 663–[66]5).

According to Tau You Wu (1986, p. 40):

Einstein's theory was criticized by P. Ehrenfest (1911–1914) [1911; 1914] and Natanson (1911) [but Natanson's article was not mentioned in bibliography], as not leading to Planck's law, but only to Wien's law. The criticism were based on the analysis of the distinction between the "indistinguishable and discrete photons" of Einstein and the "energy steps" in Planck's theory.

According to Alexander Bach (1990, pp. 24–25):

The question concerning the statistical foundations of Planck's law left open by Lorentz was answered by Natanson [Natanson 1911c – in M.K.'s notation], who explicitly referred to the contributions of Boltzmann [...] and Lorentz [...]. Compared to the work of his predecessors (except Boltzmann) and followers, Natanson's work was distinguished by his unusual precision in terminology and by his explicit determinations of probability distributions. Because Natanson fixed, as Boltzmann did, the maximum energy of any molecule, and because he met the same difficulties as did Boltzmann in taking the limit $n, d \rightarrow \infty$, Natanson directly followed Boltzmann's method of 1877. [...] Finally, using an entropy expression which is equivalent to Boltzmann's (but inserting Planck's constant k) [...], Natanson [...] obtains, by following Planck's strategy, [...], Planck's radiation formula. [...] Natanson also provides

Gleichwahrscheinliche Fälle waren die Besetzungszahlen der Oszillatoren. Natanson sah darin (1911) eine mögliche Statistik nichtunterscheidbarer Teilchen. Bose wandte die gleiche Statistik (1924) auf Lichtteilchen an, Einstein auf Gasmolekeln: er zeigte, daß die Schwankungen in einem solchen Gase sich so verhielten, als kämen sie Teilchen und von Wellen her" (Hund 1967, pp. 153–154).

a careful analysis of the difference between Bose-Einstein statistics and Maxwell-Boltzmann statistics [...]²⁶.

According to Alexei Kojevnikov (2002, p. 198):

If quantized waves can be regarded as a quantum mutation of classical waves, one could similarly try to modify

²⁶ An anonymous reviewer of this article *rightly* noted that this quotation of A. Bach “says nothing about what Natanson did differently from Boltzmann and Planck. In particular, it is not explained, why Boltzmann did not obtain Planck’s distribution before making the energy of the molecules to be continuous. (As a matter of fact, Planck’s complexions are different from Boltzmann’s complexions)”.

Answering this remark, it is worth mentioning here that L. Boltzmann (1868; 1872; 1877) considers a *system of n fictitious physical molecules, which is not realized in any mechanical problem*, since each of these molecules can take only a discrete velocity ($0/q, 1/q, 2/q, 3/q, \dots, p/q$) and a discrete alive force [i.e. twice of kinetic energy] ($0\epsilon, 1\epsilon, 2\epsilon, 3\epsilon, \dots, p\epsilon$). Moreover, w_0 molecules have a 0ϵ alive force, w_1 molecules have $2\epsilon, \dots, w_p$ molecules have $p\epsilon$, $w_0 + w_1 + w_2 + \dots + w_p = n$, and the total alive force of this system is: $0w_0 + 1w_1 + 2w_2 + \dots + pw_p = \lambda$. These assumptions are very useful to perform mathematical calculations to get the probability of state distributions of system of n such molecules. This probability is the number B of complexions in which w_0 molecules have a 0ϵ alive force, w_1 molecules have 2ϵ , etc, divided by the number of all complexions J : $B = n! / [(w_0)!(w_1)!\dots(w_p)!]$ – it means that molecules are distinguishable, and $J = (\mu \lambda + n - 1, \lambda)$ [it is a binomial coefficient, and $\mu = \lambda\epsilon/n$ (mean alive force of a molecule)] = $\lambda^{n-1} e^{n-1} / [(2\pi)^{1/2}(n-1)^{n-1/2}]$. Then, at a final step of calculation Boltzmann applies the limit of p, q and P to infinity, and ϵ to 0. Such a limit has a physical sense and leads to Maxwell-Boltzmann statistics. Cf. Boltzmann 1868 (reprinted 1909a, pp. 49–96; translated into English and commented 2014, pp. 139–142, 142–148); 1972 (reprinted 1909a, pp. 316–402); 1877 (reprinted 1909b, pp. 164–223; 2002 (English translation by Joël Le Roux)); Gallavotti 2014, pp. 178–181 (partial translation and comments by Giovanni Gallavotti); Bach (1988; 1990); Badino 2009; 2015; Nauenberg 2016, pp. 717–718.

Regarding Planck’s approach, he accepted Boltzmann’s combinatorial approach, but instead of fictitious physical molecules, he considers material entities having discrete value of energy ($0\epsilon, 1\epsilon, 2\epsilon, 3\epsilon, \dots, p\epsilon$). More about the derivation of Planck’s distributions, including his combinatorial considerations, see e.g. Rosenfeld 1936; 1958; Klein 1962; 1965; Kuhn 1978; Darrigol 2000, pp. 3–21/2001; 2003; Boya 2003; Badino 2009; 2012; 2015; Gearhart 2010, pp. 95–117; Nauenberg 2016; Passon, Grebe-Ellis 2017, pp. 5–6.

Regarding Natanson’s approach, following the idea of Boltzmann’s approach, he improved Planck’s approach and explained the idea of distinguishability and indistinguishability of material entities. Cf. Natanson 1911a; 1911c; Bergia 1987, pp. 233–235 (reprinted: 2009, pp. 343–345); Lange (1992a; 1992b; 1997a; 1997b); Spalek (2005; 2006; 2009).

somewhat the model of light quanta in order to bring it in correspondence with the Planck law. The existing contradiction, which amounted to differences in statistics, was clarified largely thanks to the efforts by Ehrenfest. He explained that statistically independent energy quanta led directly to the Wien law, while in order to obtain the Planck law, one had to assume that quanta were not independent (in the classical sense of the term, which was then the only available one) but indistinguishable objects (Ehrenfest 1911). This peculiarity of Planck's combinatorics was also understood around the same time by Ladislas Natanson (Natanson 1911 [1911c]) and a few years later explained with ultimate clarity by Ehrenfest and Kamerlingh Onnes (1915), who formulated the statistics of indistinguishable objects in comparison with the statistics of independent, or distinguishable objects in exactly the same way in which contemporary textbooks explain the difference between the Bose–Einstein and Boltzmann statistics. [...] Their understanding, however, did not immediately become part of the common knowledge in the field, which led, in particular, to further polemics in 1914, between Mieczysław Wolfke and Iurii Aleksandrovich Krutkov (Wolfke 1914a and 1914b, Krutkov 1914a and 1914b).

According to Peter Enders (2007, p. 87):

Contrary to Einstein's results, Ehrenfest (1880–1933) (cf. Ehrenfest 1911) and Natanson (1864–1937) (cf. Natanson 1911a; 1911c) explained the difference between the classical and quantum radiation laws by means of different counting rules for distinguishable and indistinguishable particles (cf. Jammer 1965, §1.4; Mehra, Rechenberg 1982, vol. 1, pt. 2, sect. V.3).²⁷

According to Magdalena Waniek and Klaus Hentchel (2011, p. 42):

Natanson had consistently furthered the turnaround already indicated by Einstein in 1905 towards an investigation

²⁷ Cf. also Enders 2009, p. 13.

of the radiation field itself, and was the first to come across the core assumption of indistinguishability.²⁸

According to Natanson's own words, this is the following idea:

in the process of estimating probabilities, the receptacles of energy *can* be treated as distinguishable and that the energy-units, being in all respects precisely alike, *cannot* be so treated. Since it is upon this assumption that our procedure ultimately rests, it seems natural to appeal to it at once as the ground work of theory. Sufficient importance does not seem to be attached to the fact that we really have no other way of demonstrating the legitimacy of Planck's method of calculating probabilities except by appealing to the experimental evidence by which the final conclusions of the calculation are supported" (Natanson 1911a, p. 139).²⁹

According to Barry R. Masters (2013, p. 43):

From a historical perspective, the little-known work [sic! – M.K.] of Ladislas Natanson is significant. He shows that both Planck and Debye have made the tacit assumption of the indistinguishability of quanta in their derivations. Both Paul Ehrenfest and Kamerlingh Onnes reach the same conclusion.

According to Oliver Passon and Johanne Grebe-Ellis (2017, p. 7):

The issue of indistinguishability in quantum theory has an exciting prehistory which is rarely mentioned in textbooks. Already in 1911 the Polish physicist Władysław (or latinized "Ladislas") Natanson scrutinized the statistical assumptions underlying Planck's law and anticipated this

²⁸ "Natanson hatte die sich bereits bei Einstein 1905 andeutende Wende hin zu einer Untersuchung des Strahlungsfeldes selbst konsequent weitergedacht und stieß dabei als Erster auf die für die statistische Ableitung eigentlich zentrale Kernannahme der Ununterscheidbarkeit" (Waniek, Hentchel 2011, p. 42).

²⁹ Waniek, Hentchel (2011, p. 42) quoted the German version of Natanson's article (Natanson 1911c, p. 662).

concept [33, i.e. Natanson 1911c]. Natanson discriminated between the situation where (i) both, the units of energy and the “receptacles of energy” can be distinguished, (ii) only the receptacles of energy can be identified (i.e., are distinguishable), or, (iii) only the units of energy are distinguishable. In either case a different combinatorics needs to be applied. Natanson claims that Planck’s equation [...] assumes the scenario (ii), i.e., treats the energy elements as indistinguishable. But he failed to draw a connection to light quanta. This connection was drawn by Paul Ehrenfest in 1911 but argued more convincingly in 1914 by Ehrenfest together with Heike Kamerlingh Onnes in the paper already quoted for the simple derivation of Planck’s combinatorial formula [20, 34, i.e. Ehrenfest 1911; Ehrenfest, Kamerlingh Onnes 1914].

And according to Klaus Hentschel (2018, p. 81):

In principle, it is possible to assign a number to each classical particle or to give it some other identifying marker because (theoretically at least) it is distinguishable from all the others. In the twentieth century when statistical mechanics was linked to the early quantum theory, it became evident that this no longer applies to the world of quanta. The indistinguishability of quantum particles stymies any attempt to identify or earmark them [...]. A Polish physicist in Cracow, Ladislas Natanson (1864–1937), was the first to realize this [...].

Thirdly, the researchers commenting Natanson’s achievements thought that Natanson and Ehrenfest were the first to understand the concept of identity of physical objects. It was claimed by the following scholars: M.J. Klein (1959); A. Pais (1986, p. 283), and P. Pesic (1991; 2012).

According to Peter Pesic (2012, p. xii):

Planck’s counting exemplifies a concept of *identity*, which joins *equality* of observable physical quantities (like mass or charge) to a radical *indistinguishability* that can confuse space-time historians (cf. Pesic 1991). This is a profound

theme of the quantum theory that Ladislas Natanson and Paul Ehrenfest were among the first to notice (1911) (cf. Pais 1986, p. 283; Klein 1959).³⁰

3.5. H. Kragh's thesis and the refutation thereof

Helge Kragh's thesis

Although the Brussels Conference on “The radiation and the Quanta” included all the key figures of quantum theory,



Fig. 9. The participants of “The radiation and quanta” Symposium. The First Solvay Conference (Brussels, 29 October – 4 November 1911). Source: http://upload.wikimedia.org/wikipedia/commons/c/ca/1911_Solvay_conference.jpg; http://en.wikipedia.org/wiki/Solvay_Conference. Seated (L-R): Walther Nernst, Marcel Brillouin, Ernest Solvay, Hendrik Lorentz, Emil Warburg, Jean Baptiste Perrin, Wilhelm Wien, Marie Curie and Henri Poincaré. Standing (L-R): Robert Goldschmidt, Max Planck, Heinrich Rubens, Arnold Sommerfeld, Frederick Lindemann, Maurice de Broglie, Martin Knudsen, Friedrich Hasenöhl, Georges Hostelet, Edouard Herzen, James Hopwood Jeans, Ernest Rutherford, Heike Kamerlingh Onnes, Albert Einstein and Paul Langevin.

³⁰ In contrast to all authors mentioned in Section 3.4, Enric Pérez Canals (2010) omits Natanson's contribution to the issue of indistinguishability in quantum theory. On the other hand, Marian Mięśowicz (1987, p. 550) overstated mistakenly that “Prof. Władysław Natanson was the first in the world to draw attention to the issue

not all of the participants were concerned with quantum problems. Two of the reports, given by Jean Perrin and Martin Knudsen, did not deal with aspects of quantum theory (Kragh 2002, p. 71).

A refutation of H. Kragh's thesis

For the historical reason given below, it is impossible to agree with the statement that the First Solvay Conference held in 1911 “included all the key figures of quantum theory”.

It suffices to repeat after A. Hermann (1971, p. 141) the list of scientists who, despite their interest in the quantum theory, were not invited to participate in the Solvay Conference. The list includes such scholars as: Arthur Erich Haas, Artur Schidlof, Ludwik Hopf, Ladislas Natanson, Peter Debye, Niels Bjerrum, Richard Gans, Pierre Weis, Emil Warburg, James Franck, Edgar Meyer, and Friedrich Paschen.

Furthermore M.J. Konieczny (2008, [2010](#); [2011](#); [2012](#)) and N. Nagasawa (“Minamida” 2009; Nagasawa [2018](#)) emphasize justly the fact that Ladislas Natanson was not invited to participate in the Conference, though he was at that time one of the best experts of the subject matter of “the Radiation and the Quanta”.

**3.6. Who, in the years 1911–1925, knew Natanson's works
on Planck's theories of radiation and related issues?
The results obtained so far by other researchers
than the author of this article**

The first researchers of this issue formulated two related theses:

- *F. Hund's thesis*: “In 1924 Natanson's arguments had been already forgotten” (Hund 1967, p. 123).
- *A. Kastler's thesis*: “A paper [Natanson 1911c – M.K.] which unfortunately remained unnoticed and unmentioned” (Kastler 1983, p. 617).

of quantum statistics as early as 1911”. („Na zagadnienie statystyk kwantowych, pierwszy w świecie zwrócił uwagę już w roku 1911 prof. Władysław Natanson”).

Then S. Bergia (1987; reprinted version 2009) made an empirical test of F. Hund's and A. Kastler's theses. He received both negative and positive results.

Negative results:

- S. Bergia (1987, p. 234; reprinted version 2009, p. 344) could not find a reference to Natanson in Planck's research papers.

Positive results:

- S. Bergia listed four scientists that read Natanson's work (1911c):
 - 1) M. Masius – The English translator of the 2nd ed. of Planck's *Vorlesungen über die Theorie der Wärmestrahlung* (1914).³¹
 - 2) George Krutkow³² (1914a) gave a reference to Natanson's paper (1911c).³³
 - 3) Mieczysław Wolfke (*although S. Bergia does not show any reference where Natanson is cited*).³⁴
 - 4) Paul Ehrenfest and Heike Kamerlingh Onnes (*although S. Bergia does not show any reference where Natanson is cited*).³⁵
 - 5) Louis de Broglie (*although S. Bergia does not show any reference where Natanson is cited*).³⁶

Then N. Nagasawa ("Minamida" 2009; Nagasawa 2018) made a subsequent empirical test of F. Hund's, A. Kastler's and S. Bergia's theses.

Negative results:

- N. Nagasawa could not find a reference to Natanson in Einstein's research papers.

Positive results:

- N. Nagasawa listed eight scientists that read Natanson's work:
 - 1) Max Planck (Solway Congress's talk, then he cited Natanson's article (1911c) in his own article published in Proceedings).³⁷

³¹ Bergia 1987, p. 234 (reprinted version 2009, p. 344).

³² George Krutkow this is Iurii Aleksandrovich Krutkow.

³³ Bergia 1987, p. 235 (reprinted version 2009, p. 345).

³⁴ *Ibid.*, pp. 235–236, 239–240 (reprinted version 2009, pp. 345–346, 349–350).

³⁵ *Ibid.*, pp. 236–239 (reprinted version 2009, pp. 346–349).

³⁶ *Ibid.*, pp. 240–243 (reprinted version 2009, pp. 350–353).

³⁷ According to N. Nagasawa ("Minamida" 2009 / Nagasawa 2018, p. 397), it was in fact a negative citation: "These calculations [made by Planck himself] are complete

- 2) Arnold Sommerfeld knew Natanson's article (1911c; published: 15 August 1911) as evidenced by Sommerfeld's letter sent to Natanson from München, dated October 3, 1911. However, he did not cite it in his works.³⁸
- 3) Paul Ehrenfest knew Natanson's article (1911c; published: 15 August 1911) as evidenced by Ehrenfest's letter sent to Sommerfeld from St. Petersburg, dated October 16, 1911.³⁹ However, he did not quote this Natanson's work in his articles.
- 4) M. [Morton] Masius – the English translator of the 2nd ed. of Planck's *Vorlesungen über die Theorie der Wärmestrahlung* (1914) mentions two Natanson's works (1911c; 1912b).⁴⁰
- 5) Friedrich Krüger, the chief editor of the *Physikalische Zeitschrift (Danzig-Langfuhr)*, knew both the English version of Natanson's article (1911a), and the German version (1911c) as evidenced by two Krüger's letters sent to Natanson from Berlin, dated April 24, 1911 and July 22, 1911.
- 6) M. Iklé, who translated Natanson (1911a) for the *Physikalische Zeitschrift* (Natanson 1911c), knew also both versions of Natanson's article.

and do not contain such uncertainty that recently Natanson described in the “Phys. Zeitschr.” (Planck 1911a). (Nagasawa repeated this translation after O. Darignol (1991, p. 254)). Moreover, according to Nagasawa: “This is the only instance that we can find of a third party referring to Natanson's paper before World War II.” I will show in the further part of the article that O. Darignol and N. Nobukata Nagasawa are wrong in both of these points.

³⁸ Cf. Nagasawa 2018, pp. 397–398.

³⁹ Cf. *Ibid.*, pp. 398–399.

Moreover, Ehrenfests doesn't mention Natanson's article in their review work finished in September 1911: Ehrenfest, Ehrenfest-Afanaseva 1911 (Supplements completed in September 1911), p. 84 fn. 237 / (English translation) 1959, p. 104, fn. 245. In this context it is cited only the article of P. Ehrenfest (1911).

⁴⁰ Planck mentioned them in 1914 in Appendix II. “References”, included in his English translation of Max Planck's *Vorlesungen über die Theorie der Wärmestrahlung* (2nd ed., 1913). We read here that the appendix gives “a list of the most important papers on the subject treated in this book and others closely related to them” and was created “with Professor Planck's permission” (see: Planck 1914, p. iv).

S. Bergia 1987, p. 234 (reprinted version 2009, p. 344) doubts that Max Planck did so, and thinks that Natanson's works were included in the appendix without the knowledge of Planck. However, Bergia does not show any evidence for this statement. In contrast, I do not doubt the words of M. Masius. I explain this in the further part of this article.

- 7) Samuel Hawksley Burbury⁴¹ – his note #344 (1911) includes only the bibliographic records of Natanson's articles: 1911a and 1911c without a review of these works.
- 8) Edwin Henry Barton⁴² – his article #733 (1912) is a long positively evaluated review of Natanson's article (1912b).⁴³
- 9) Japan physicists Hantaro Nagaoka (1865–1950) and Jun Ishiwara (1881–1947) could have known and read Natanson's article (1911a), because the former had a copy of this article (and five other Natanson's works published in Kraków from 1904 to 1931), and the latter because he used to be a student of the former. Nevertheless, they do not cite the Natanson's article in their works, particularly in the article of Ishiwara (1911–1912; in German, and published in Tokyo).⁴⁴

4. Who knew Natanson's works on Planck's theory of radiation and related issues in the years 1911–1925? The results of the author of this article

Following the footsteps of A. Kastler, S. Bergia and “R. Minamida”/N. Nagasawa, I looked in my research for recipients of Natanson's works on the subject under discussion. First of all, I analysed once again the course and content of two conferences of 1911: the first in Kraków and the second in Brussel. Then I studied Natanson's correspondence and I sought the publications that cited Natanson's works.

4.1. 11th Congress of Polish Physicians and Natural Scientists in Kraków (18 – 22 July, 1911) – Einstein, Natanson, Smoluchowski, and Olszewski

At the beginning of 1911 Albert Einstein intended to participate in the 11th Congress of Polish Physicians and Natural Scientists in Kraków

⁴¹ Samuel Hawksley Burbury (1831–1911), a British mathematician and physicist. He died on 18 August 1911. According to “Minamida” 2009 / Nagasawa [2018](#) (pp. 407–408) it was the reason that Natanson's work was not reviewed by Burbury. I think it is a very probable explanation.

⁴² Edwin Henry Barton (1859–1925) was professor of experimental physics at University College, Nottingham.

⁴³ Cf. “Minamida” 2009/Nagasawa [2018](#), p. 408.

⁴⁴ Cf. *ibid.*, pp. 399–402.

(18–22 July, 1911) but he finally decided against this idea. We know that from the draft letter written by Einstein in Prague before 21 July 1911 and sent to the Institute of Physics of the University of Kraków directed by nobody else but the protagonist of this paper Władysław Natanson⁴⁵

The received message was paraphrased in *Dziennik XI. Zjazdu Lekarzy i Przyrodników Polskich w Krakowie* (“The Daily Proceedings of Polish Physicians and Natural Scientists in Kraków”), No. 4, 21 July 1911, p. 4 (see below).

Znakomity uczony prof. Einstein z Pragi nadesłał bardzo serdeczną depeszę od [do – M.K.] sekcji fizycznej.

The illustrious scientist professor Einstein sent from Prague a heartfelt message to the physical section {of the 11th Congress of Polish Physicians and Natural Scientists} [translation – M.K.].

During this 11th Congress of Polish Physicians and Natural Scientists, on 19 June, 1911, the 2nd General Session of the Section of Exact Sciences held talks of two scholars: M. Smoluchowski, “Atomistyka współczesna” (“Contemporary atomics”), W. Natanson, “O promieniowaniu” (“On radiation”). After these talks Karol Olszewski gave an additional lecture on cryogenic instruments combined with an exhibition of these instruments.

We know that all these three speakers were eminent scientists,⁴⁶ and that all the three talks were highly evaluated by over 100 participants of the section of exact sciences. It is evidenced by the report from these events in *Dziennik XI. Zjazdu Lekarzy i Przyrodników Polskich 1911*, [No. 3](#), p. 1:

⁴⁵ See document 273 in German, in: *The Collected Papers of Albert Einstein*, vol. 5, *The Swiss Years: Correspondence, 1902–1914*, p. 306; and its English translation by Anna Beck in: Einstein 1995, p. 195. That fact was discussed by Nobukata Nagasawa (“Roh Minamida” 2009, p. 3), however without mentioning the source of the information; he did so later (Nagasawa 2018, pp. 396–397).

⁴⁶ Smoluchowski deserved the Nobel Prizes in chemistry and physics, but he did not receive them, because he died in 1917. He should have received it in 1925 in chemistry together with an Austrian of Hungarian origin, Richard Zsigmondy, professor at the University of Göttingen, and / or in 1926, together with Teodor Svedberg, Swedish professor at the University of Uppsala, whose experimental work on colloids was closely

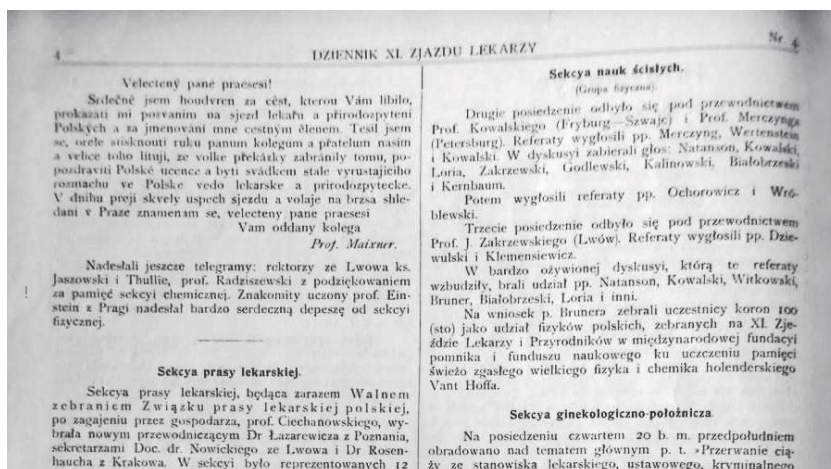


Fig. 10. *Daily Proceedings of the Congress of Polish Physicians and Natural Scientists in Cracow*, No. 4, 21 July 1911, p. 4. Source: © Biblioteka Jagiellońska; photo: © Michał Kokowski.

The meeting was held under the chairmanship of prof. Dickstein from Warsaw with a large audience (over 100 people). The talks of prof. Smoluchowski from Lviv and

related to Smoluchowski's theoretical work. He should have also received it in 1926 in physics, together with Jean Perrin, French professor at the University of Sorbonne in Paris for experimental work on the Brownian motion, confirming the validity of the molecular theory of Einstein-Smoluchowski. Cf. Zsigmondy [1926](#); Perin [1926](#); Svedberg [1927](#); Crawford, Heilbron, Ulrich 1987; Wróblewski [2012](#); The Nobel Prize [2019](#).

Olszewski was nominated for the Nobel Prize in physics twice: in 1904 (nominator: [Nikolay Umov](#) from Moscow State University) and in 1913 (nominators: [Ladislav Natanson](#), [August Witkowski](#), [Constantin Zakrzewski](#), [Maurycy Rudzki](#), all from the Jagiellonian University in Kraków), and once for the Nobel Prize in chemistry in 1913 (nominator: [Karl Dziewulski](#) from the Jagiellonian University in Kraków) – cf. Crawford, Heilbron, Ulrich 1987; Wróblewski [2012](#); The Nobel Prize [2019](#). Why Olszewski did not receive this award is a mystery to me, which can be explained on the basis of the analysis of external factors in science (i.e. international politics and sociology).

Natanson was also a very good physicist. In this article I focus on showing the reception of Natanson's achievements in statistics of radiation and related themes. Regarding the general description of Natanson's achievements in physics, cf. Weysenhoff [1937](#) (pp. 289–294 (in English)); [1958](#) (pp. 120–124 (in Polish)); Średniawa 1985, pp. 89–90, 116–117; 2000, pp. 447–458; 2001, pp. 104–107. Regarding Natanson's achievements in theory of irreversible and reversible phenomena, cf. Kokowski [1993](#); [1994](#); [1997](#).



Fig. 11. Marian Smoluchowski, circa 1914.
Source: <http://web.archive.org/web/20051227102418/http://www.zwoje-scrolls.com/zwoje35/portret-krakow.jpg>



Fig. 12. Władysław Natanson, circa 1910.
Source: *Władysław Natanson 1864–1937* (2009), illustration 14.



Fig. 13. Karol Olszewski, “A king of low temperatures”, circa 1900. Source: *Wikipedia* (https://commons.wikimedia.org/wiki/File:Karol_Olszewski.jpg).

Natanson from Kraków, touching on the deepest issues of contemporary physics and chemistry, made a deep impression on the listeners and met with the warmest of receptions.

The meeting was continued at the First Chemical Laboratory of the Jagiellonian University, where prof. Olszewski arranged an exhibition of equipment used to liquefy gases, mostly of their own construction. In his lecture, the lecturer gave an outline of his classical research and, in addition, demonstrated his latest instruments for separating oxygen and nitrogen from liquefied air, as well as a device for simultaneous condensation of air and hydrogen.

The exhibition arranged by prof. Olszewski with a great deal of work casts a bright light on the development of these studies, whose fruits played such an outstanding role in the development of science.⁴⁷

Hence, there is no doubt that over 100 Polish physicists and chemists participating on 19 June, 1911 in Natanson's talk during the 11th Congress of Physicians and Naturalists knew at least one Natanson's publication on the statistics of radiation (Natanson 1911b).

Moreover, we know from Natanson's correspondence that long before this congress, on 22 December 1910, Natanson shared with Smoluchowski his view on problem of Planck's theory of radiation:

⁴⁷ "Posiedzenie odbyło się pod przewodnictwem prof. Dicksteina z Warszawy przy bardzo liczny udziałem słuchaczy (przeszło 100). Odczyty prof. Smoluchowskiego ze Lwowa i Natansona z Krakowa, poruszające najgłębsze zagadnienia współczesnej fizyki i chemii wywarły na słuchaczach głębokie wrażenie i spotkały się z gorącym uznaniem.

Dalszy ciąg posiedzenia miał miejsce w I. pracowni chemicznej uniwersytetu, gdzie prof. Olszewski urządził wystawę aparatów, przeważnie własnej konstrukcji, służących do skraplania gazów. W swym referacie, podał prelegent historyczny zarys swych klasycznych badań a oprócz tego zademonstrował najnowsze swe przyrządy do oddzielania tlenu i azotu ze skroplonego powietrza, jako też przyrząd do równoczesnego skraplania powietrza i wodoru.

Wystawa urządzona przez prof. Olszewskiego z wielkim nakładem pracy, rzuci jasne światło na rozwój tych badań, których owoce tak wybitną rolę odegrały w rozwoju nauki" (*Dziennik XI. Zjazdu Lekarzy i Przyrodników Polskich 1911, No. 3*, p. 1).

Dear Sir, [...] I look [...] forward to your announced Congress speech ["Contemporary Atomics"]. Doctor Zakrzewski insists on me to speak of "atomics in energetics" in a similar manner and I almost agreed; but perhaps I will be relieved of the promise if "quanta" disappear as quickly as it has been in science so far; until July, there may not be any atomics in energetics and I will have nothing to talk about. It is a beautiful theory, the one of radiation; it's just a shame that what is solid and perfectly justified, i.e. equipartition, does not agree with the facts, and Planck, who agrees, seems to have no basis for that at all! It seems to me that Planck, even in his own way, should have received $R\nu\lambda^{-4}$ (Rayleigh-Jeans law) and I cannot understand why his formula is true. These are very difficult *problemata* [translated by M.K.].⁴⁸

In another letter of 26 April 1911 r. Natanson writes to Smoluchowski:

We have already been thinking for a few months with Mr. Zakrzewski about applying to metals – instead of the usual equipartition Maxw. [Maxwellian] theory of electron motion – the Einsteinian-Planckian theory of $(e\varepsilon/k\nu - 1)$ etc. Well, it's so difficult to concentrate. There is still a lot to do in the matter itself; we are not quite ready yet⁴⁹ [translated by M.K.].

⁴⁸ "Cieszę się [...] z zapowiedzianego referatu Szan[ownego] Pana na Zjeździe. Dr. Zakrzewski nalega na mnie, abym mówił podobnie o «atomistyce w energetyce» i niemal że [niemalże] się zgodziłem; lecz może będę zwolniony zobietnicy [z obietnicy], jeżeli «quanta» będą znikaly równie prędko, jak dotych czas [dotychczas] z Nauki; do Lipca [lipca] może Atomistyki w Energetyce już wcale nie będzie i nie będę miał o czym mówić. Przepiękna to jest teoria, ta Promieniowania; tylko szkoda, że to, co jest mocne i znakomicie uzasadnione, ekwipartyca [ekwipartycja], nie zgadza się z faktami, a Planck, który się zgadza, zdaje mi się, że nie ma wcale podstaw! Tak mi się wydaje, że Planck, nawet na swojej własnej drodze powinien był otrzymać $R\nu\lambda^{-4}$ (p. [prawo] Rayleigha-Jeansa), i nie mogę zrozumieć, czemu jego wzór jest prawdziwy. Bardzo są to trudne problemata" (A letter from Władysław Natanson to Marian Smoluchowski, Kraków, 22 December, 1910; transcription – M.K.; see Natanson 1910 (*archival document*), folium 60 verso).

⁴⁹ "Z p. Zakrzewskim już od kilku miesięcy myślimy o tem [tym], że do metali zastosować, zamiast zwykłej ekwipartycyjnej Maxw. [Maxwellowskiej] teorii ruchu elek-

Hence, it is certain that Marian Smoluchowski knew Natanson's views on Planck's theory of radiation and the related Einstein's issues also directly from their private correspondence.

Moreover, because Albert Einstein (1879–1955) was to be a participant of the 11th Congress for Polish Physicians and Natural Scientists (Kraków, 18–22 July 1911), he was probably informed about the planned lectures of Smoluchowski, Natanson, and Olszewski, as well as on Natanson's first article (1911a; published circa 10 April 1911). However, so far, there are no historical documents that would prove this hypothesis.

4.2. The First Solvay Conference (Brussels, 29 October – 4 November 1911) – Planck, Natanson, and Einstein

During the First Solvay Conference held “The radiation and quanta” Symposium. Among participants of this meeting were also Max Planck, Arnold Sommerfeld, Paul Ehrenfest and Albert Einstein. Planck had a talk entitled: “La loi du rayonnement noir et l’hypothèse des quantités élémentaires d’action” (“The law of black radiation and the hypothesis of elementary quantities of action”).⁵⁰

At that time or a bit later Planck not only knew Natanson's article (1911c), but also understood its essence, as evidenced by his remark in his article (1912) published in the Proceeding of this conference:

Ce calcul ne prête à aucune ambiguïté et ne renferme en particulier plus rien de l'indétermination dont L. Natanson a récemment parlé dans le *Phys. Zeitschr.*, t. XII, 1911 [i.e. Natanson 1911c], p. 659 (Planck 1912, p. 104, fn. 1).⁵¹

To avoid doubts, let us to cite its equivalent German and English translations:

tronów – Einstein Plankowską teorię z ($e^{\epsilon/kv} - 1$) etc. Cóż kiedy tak trudno o możliwość skupienia się. W samej kwestyi, jest jeszcze dużo do zrobienia; nie jesteśmy jeszcze w zupełnym porządku” (Natanson 1910 (*archival document*), folium 71 recto).

⁵⁰ Cf. Solvay et al. 1912, pp. 93–114, 115–132.

⁵¹ To my knowledge, it is the first citation of a German version of Natanson article (1911c), and the second citation after Zakrzewski (1911, p. 329 fn. 1), who cited the English version of Natanson's article (1911a).

Diese Berechnung ist vollkommen eindeutig und enthält insbesondere nichts mehr von der Unbestimmtheit, welche L. Natanson neuerdings mit Recht zur Sprache gebracht hat (translated by Arnold Eucken 1914; cited by Lange 1992, p. 22, and by Straumann 2011, p. 12).

This calculation is completely unambivalent and in particular no longer contains the indefiniteness about which L. Natanson has recently spoken with justification (firstly translated by Stahel 2000, p. 246; repeated by Straumann 2011, p. 12).⁵²

Hence, it is certain that Planck appreciated Natanson's article (1911c),⁵³ and many scientists, including Einstein, could read Planck's article and learned about the existence of Natanson's article.

4.3. List of the scientists who knew and appreciated Natanson's works (author's results)

- 1) Marian Smoluchowski (1872–1917), professor of the Lviv University (1900–1912) and professor of the Jagiellonian University (1912–1917), a friend of Natanson, knew about his first objections regarding Planck's considerations⁵⁴ and that Natanson together with Zakrzewski planned to extend "the Einstein-Planck theory ($e^{\varepsilon/kv} - 1$) etc. to metal"⁵⁵; Smoluchowski knew

⁵² This English translation correctly reflects the content of the French original and its German translation – please compare them with the English translation provided by Nobukata Nagasawa ("Minamida" 2009 / Nagasawa 2018, p. 397) – see fn. 37.

More about the first conference, cf. Mehra 1975, pp. 12–72 (pp. 24–40, about Planck lecture).

⁵³ Moreover, it was Max Planck, who supported the nomination of Władysław Natanson to the Deutscher Physikalischen Gesellschaft at the end of January 1913, see Appendix 3. It is obvious that he would not have done it, if he had not valued Natanson's scientific achievements.

Therefore, in contrast to S. Bergia (1987, p. 234; reprinted version 2009, p. 344) – cf. fn. 40 above – I think that M. Maius included in the Appendix II two Natanson's works (1911c, 1912b) with the knowledge of Planck himself.

⁵⁴ The letter of 22 December 1910 from Władysław Natanson to Marian Smoluchowski (see Natanson 1910 (*archival document*), folium 60 verso).

⁵⁵ The letter of 26 April 1911 from Władysław Natanson to Marian Smoluchowski (see Natanson 1911c (*archival document*), folium 71 recto). Natanson considered this issue in his article of 1912 and Zakrzewski in his article of 1911.

Natanson's articles: 1911a; 1911b; 1911c; and probably also other Natanson's works excluding article of 1924b (since he died in 1917).⁵⁶

- 2) Władysław Gorczyński (1879–1953), a Polish meteorologist from the Meteorological Office at the Museum in Warsaw, knew at least the first Natanson's article (Natanson 1911a).⁵⁷
- 3) Konstanty Zakrzewski (1876–1948), professor of the Jagielloonian University (1911–1913, and since 1917) and professor of the Lviv University (1913–1917), Natanson's college, a co-organizer of the Section of Exact Sciences during the 11th Congress for Polish Physicians and Natural Scientists (Kraków, 18–22 July, 1911),⁵⁸ knew at least six works (Natanson 1911a; 1911b; 1911c; 1912a; 1912b; [1913](#)); he quoted the first Natanson's work (1911a) in his own article (Zakrzewski 1911, received: 3 April 1911, p. 329 fn. 1)⁵⁹; he co-operated with Natanson and planned to extend the Einstein-Planck theory to metal;⁶⁰ in a letter on 23 January 1916, he discussed with Natanson a plea of plagiarism against George Jaffé's article from *Annalen der Physik* ([1914](#)) about optical properties of metals that repeated many formulas published earlier in Zakrzewski's article in German (1911a), which also cited Natanson (1911a; received: 6 March 1911; published: circa 6 April 1911);⁶¹ and then he

⁵⁶ In the letter of 30 March 1911 (see Natanson 191b (*archival document*), folium 69 verso), Natanson informed Smoluchowski that he would send in a week or ten days the article on Radiation from the *Bulletin* (i.e. 1911a). On 19 June 1911 in Kraków, Smoluchowski attended Natanson's lecture on radiation – cf. above Section 4.1, so he most likely knew also at least the text of the lecture (Natanson 1911b) and possibly also its reprint (1912a).

⁵⁷ In the letter of 10 April 1911 (Gorczyński 1911 (*archival document*)), he thanked Natanson for sending a copy of this article. About Gorczyński's career, see Śródka [1983](#).

⁵⁸ Cf. *Dziennik XI. Zjazdu Lekarzy i Przyrodników Polskich w Krakowie* (*Daily Proceedings of Polish Physicians and Natural Scientists in Cracow*), No. 1, 21 July 1911, p. 10.

⁵⁹ To my knowledge, it is the first citation of Natanson's article (1911a) in literature, but according to N. Nagasawa only Max Planck cited in 1912 Natanson's work on radiation and related matters (1911c) – cf. fn. 37 above.

⁶⁰ See fn. 55.

⁶¹ Zakrzewski 1916 (*archival document*).

published this objection in *Annalen der Physik* (Zakrzewski 1916, received: 10 February 1916).

- 4) A group of over 100 Polish physicists and chemists (including Smoluchowski, Zakrzewski) that on 19 June, 1911 participated at the 2nd General Session of the Section of Exact Sciences during the 11th Congress for Polish Physicians and Natural Scientists which was held between the 18th and the 22nd of July in 1911 in Kraków. They heard the Natanson's lecture entitled "O promieniowaniu" ("On radiation"), published as Natanson 1911b, and also the Smoluchowski's lecture "Atomistyka współczesna" ("Contemporary atomics"), published as Smoluchowski 1911.⁶²
- 5) Jean Becquerel (1878–1953) could know Natanson 1911a – he received this work from Natanson himself on which informs the handy annotation of Natanson.⁶³
- 6) Hugo von Seeliger (1849–1924) informed about Natanson 1911a, 1912b in his notes in *Jahrbuch über die Fortschritte der Mathematik* (1912, 1913).
- 7) Arnold Sommerfeld knew Natanson's article (1911c; published: 15 August 1911) as evidenced by Sommerfeld's letter sent to Natanson from München, dated October 3, 1911 – see Sommerfeld 1911 (*archival document*).⁶⁴

However, he did not cite it in his works, including Sommerfeld 1911a (a paper given on 25 September 1911 in Karlsruhe during

⁶² See Section 4.1. It is linked with an evaluation of quality of Polish physics in 1910s.

⁶³ The work belonged to the Library of Becquerel family. It could be bought in 2011 in one antiquarian bookstore.

⁶⁴ The fact was noticed by N. Nagasawa 2018, who quoted and translated an appropriate excerpt of this letter into English: "[...] Ich bin Ihnen aufrichtig dankbar, dass Sie mir regelmässig Ihre sehr interessanten Arbeiten zusenden, die ich stets genau verfolge; ich werde mich bald mit meinem Carlsruher Vortrag über Quantentheorie und einigen Anderen revangieren. [...]" (cited by Nagasawa 2018, p. 397). In English translation: "[...] I sincerely appreciate that you regularly send me your very interesting works, which I always thoroughly follow; I will soon return the favour with my Karlsruhe lecture on quantum theory, and some other papers. [...]" (Nagasawa 2018, p. 398).

On A. Sommerfeld's scientific biography, cf. Eckert 2013a (German) / 2013b (English).

the 83rd Meeting of the German Natural Scientists and Medical Doctors Association) and Sommerfeld 1911b.⁶⁵

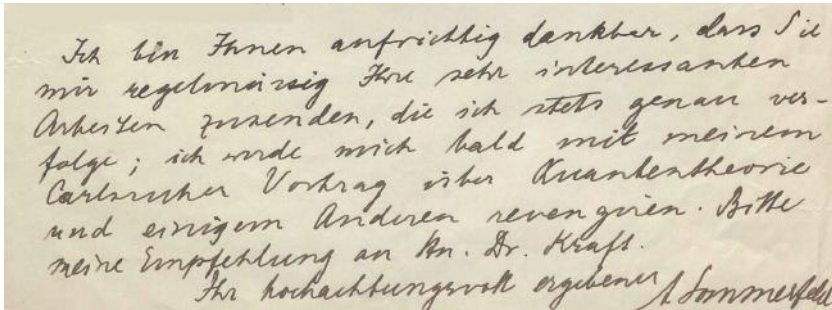


Fig. 14. The excerpt of Sommerfeld's letter sent to Natanson from Munich, dated October 3, 1911. Source: Sommerfeld 1911 (*archival document*), folium 107 recto. Photo: © Michal Kokowski.

- 8) Paul Ehrenfest (1880–1933), who worked in Saint Petersburg Polytechnic Institute between the summer of 1907 and September 1912, and then succeeded Hendrik Antoon Lorentz (1853–1928) in the chair of theoretical physics at the University of Leiden in 1912, knew Natanson's article (1911c) and discussed about it with Sommerfeld in the letter of 16 October 1911.⁶⁶

⁶⁵ On the ground of rational and ethical argumentation, I do not understand why A. Sommerfeld did not cite Natanson's works. Perhaps other, irrational reasons had to decide on their attitude to Natanson. Cf. Sections 5 and 9.

⁶⁶ Ehrenfest sent from St. Petersburg a letter dated 16 October, 1911 to Sommerfeld (Ehrenfest 1911b (*archival document*)) that showed that both knew Natanson article. The fact was noticed by "Minamida" 2009 / Nagasawa 2018, pp. 398–399, who quoted an appropriate excerpt of the letter and translated it into English.

Archiv: [München, DM](#) (Archiv HS 1977-28/A,76): "Die Bemerkungen, die kürzlich Nathanson [sic!] über die combinatorischen Grundlagen der Planckschen Theorie publiziert hatte ich ebenfalls gefunden und vor dem Erscheinen der Arbeit von Nathanson in der hiesigen physikalischen Gesellschaft vorgetragen. Aber Nathanson hat die Lösung der Schwierigkeit nicht gefunden: er hat eben nicht bemerkt, daß die Planckschen und Einsteinsche Hypothese total verschieden sind" (cited by Nagasawa 2018, p. 398). In English translation: "Remark: I had also found the recent publication of Nathanson on the combinatorial foundations of Planck's theory. I had presented [the idea] at the local physical society before Nathanson's paper was published.

However, he did not quote this Natanson's article or other works of the author in his own works.⁶⁷

- 9) Max Planck (1858–1947) not only knew Natanson's article (1911c) and understood its essence, but also appreciated it. Planck stated so clearly in his paper in the Proceedings of the First Solvay Congress (held from 30 October to 3 November 1911) – cf. Planck [1912](#), p. 104, fn. 1. So, the other participants of the First Solvay Congress, including Albert Einstein, Arnold Sommerfeld, and Paul Ehrenfest, had to know about Natanson's article (1911c). Moreover, at the end of January 1913, it was Planck who supported Natanson's efforts to become a member of the German Physical Society and recommended his candidature.⁶⁸
- 10) Iurii A. Krutkov (1890–1952), a student of Paul Ehrenfest in Saint Petersburg Polytechnic Institute (i.e. 1908–1912), and his co-worker in Leiden University (from the summer months of 1913 to the beginnings of March of 1914),⁶⁹ published two papers: Krutkov⁷⁰ 1914a (received 6 January 1914) and 1914b (received 8 March 1914). The former article mentions Natanson 1911c on pp. 134 and 136, and the latter – Natanson 1911a and 1911c on p. 363.⁷¹ Moreover, the former article states that

But Nathanson did not find the solution to the difficulty: he did not notice that Planck's and Einstein's hypotheses are totally different" (translated by Nagasawa [2018](#), pp. 398–399).

On P. Ehrenfest scientific biography, cf. Klein 1985; Huijnen, Kox [2007](#). Regarding Ehrenfest's views on statistics of radiation and related matters, cf. also Navarro, Pérez 2004.

⁶⁷ This is the same behavior as in the case of A. Sommerfeld. Hence, fn. 64 should be referred also to P. Ehrenfest.

⁶⁸ Cf. Appendix 3.

⁶⁹ On Krutkov's biography cf. Frenkel [1970](#); *Encyclopedia.com* [2019](#).

⁷⁰ Please note that I.A. Krutkov's works are mentioned in the bibliography under the name "George Krutkov" because he used this form in his articles published in the German journals.

⁷¹ To my current knowledge and in contrast to N. Nagasawa (cf. fn. 37 above), it is the second (after Planck 1912) citation of the German version of Natanson's article (1911c); and the second citation (after Zakrzewski 1911) of the English version (1911a).

Konstanty Zakrzewski informed Natanson about Krutkov's article (1914a) in the letter on 10 February 1914 (Zakrzewski 1916 (*archival document*), folium 52 verso –

Planck's assumption of independent light quanta leads to Wien's radiation formula, and the latter that Wolfke's assumption of the light atoms leads to the same Wien's radiation formula, and the basis of these conclusions were combinatorial and statistical considerations made earlier by Ehrenfest [1911a](#) (received: 8 July 1911) and Natanson 1911a (presented: 6 March 1911; published circa 10 April 1911); 1911c (received: 29 April 1911; published: 15 August 1911).

- 11) Krutkov's articles were well-known by scientists, and Ehrenfest contributed himself to this: he firstly informed Hendrik Antoon Lorentz (1853–1928) about Krutkov's article (1914a).⁷² Then together with Kamerlingh published an article (Ehrenfest, Kamerlingh [1914](#); communicated in the meeting of 31 October 1914; published on December 31, 1914; English translation [1914](#)) where two Krutkov's works are cited (1914a; 1914b).
- 12) It is very probable that Ehrenfest, who had known Natanson's article (1911c), contributed to that his student Krutkov knew Natanson's articles (1911c; 1911a). In the opposite, and less probable case, the second Krutkov's article (1914b) resulted in Eherenfest also knowing about the existence of the primary English version of Natanson's article (1911a).⁷³ Regardless of

folium 53 recto): "Przrzucalem natomiast w ostatnim «Physik. Zeitsch.» rozprawę Krutkowa [1914a] i cieszę się, że Pańska praca o promieniowaniu znalazła uznanie. Choć doprawdy od promieniowania i quantów głowa już puchnie: coraz to coś nowego, a stare wątpliwości wcale się nie zmniejszają". In an English translation: "I was browsing Krutkov's article in the last «Physik. Zeitsch.» [1914a] and I am glad that your work about radiation has been appreciated. Though indeed radiation and quanta already give me headaches: there is more and more of the new, and old doubts are not reduced at all" [transl. – M.K.].

⁷² Cf. the letter from Ehrenfest to Lorentz (24 January 1914), cited by Darrigol 1991, p. 255, fn. 21, and see also Lorentz [2018](#), Doc. 155, pp. 392–395.

O. Darrigol described Krutkov as "a student of Einstein's in Leiden." However we know that Krutkov was an assistant of Ehrenfest already in St. Petersburg, and he was in Leiden in 1913/1914 (cf. Lorentz 2018, p. 395, fn. 5; Frenkel' [1970](#); van Lunteren [2003](#)).

⁷³ Ehrenfest had a copy of Krutkov's work (1914b), since on its back he wrote a draft of a letter to Albert Einstein (regarding his answer to Einstein's comment about rotating electrons in a magnetic field) – see: Einstein 1998a, p. 15, [Doc. 4](#) (English translation) Einstein 1998b, p. 11, [Doc. 4](#).

that Ehrenfest did not quote Natanson's publications on radiation statistics and related issues in his own publications.⁷⁴

- 13) Mieczysław Wolfke, a privat-docent in the ETH (1913–1914), a docent in the University of Zurich (1914–1922), and then a professor at Warsaw Polytechnic, and a colleague of Einstein from ETH (where Einstein was a professor), did not cite Natanson's works in a series of his articles from 1913 to 1914.⁷⁵ Nevertheless he knew about the existence of Natanson's works on radiation (and related subjects), at least indirectly by reading the articles of Krutkov (1914a; 1914b – this is a polemic with Wolfke). Moreover, Wolfke knew also a revised version of Natanson's article 1911b, i.e. 1924b because in the letter of 3 September 1924, he thanked Natanson for sending him Natanson's book (1924a) which included the article – see Wolfke 1924 (*archival document*).⁷⁶
- 14) Max Born (1882–1970)⁷⁷ and Theodore von Kármán (1881–1963) not only knew Natanson's article (1912b), but they

⁷⁴ It is evidently a negative behaviour. I will consider this problem later.

⁷⁵ It is a contrary view to S. Bergia (1987, p. 235; repr. 2009, p. 345).

⁷⁶ About Wolfke's views on "atoms of light", see Mehra, Rechenberg 1982, p. 559; Bergia 1987, pp. 235–236, 239–240 (reprinted version 2009, pp. 345–346, 349–350); Howard 1990, pp. 76–78.

⁷⁷ Max Born was a German Jew, born in Breslau (Wrocław), converted to Lutheranism in 1913 (cf. G.V.R. Born 2002) and, according to some of his biographers, until 1933 (or a bit earlier) he was an advocate of German nationalism or Pan-German chauvinism (cf. Wolff 2003, pp. 337–338; Bromberg 2006). One example of such attitude of 23 November 1914 and 23 February 1915: "The *Physikalische Zeitschrift* printed the names of colleagues fighting at the front, and of those who had been decorated, wounded, or killed. Photographs framed in black accompanied obituaries of the fallen. One of the journal's editors, Max Born, explained that it wanted to demonstrate to foreign countries that «physics too is at one with the fatherland in this time of peril and danger»" (Wolff 2003, pp. 337–338).

However, later he was a member of "Vereinigung Gleichgesinnter" (Association of People with the Same Opinion) founded on 8 June 1916. It was "a confidential discussion group of pacifist intellectuals, whose aim was to make the theoretical discussions concerning internal and foreign politics [...] in the spirit of overcoming unethical nationalism and of replacing power politics by politics based on ethics, and [...] to influence the press and to reestablish international relations with scientists abroad as soon as this might become possible again" (Goenner, Castagnetti 1996, pp. 26–27). Then, with the development of Nazi anti-Semitism in Germany, the loss

mentioned this fact in their paper (1913a) in the context of priority of the solution to the problem of specific heats of crystal solids by Natanson, Debye, and Born and von Kármán.⁷⁸ Hence, Max Born and Theodore von Kármán knew Natanson's work (1912b), as well as they could know his earlier articles (1911a and 1911c).

- 15) Czesław Białobrzęski (1878–1953), a privat-docent in the University of Kiev (1907–1913), a docent in the University of Kiev (1913–1919),⁷⁹ a profesor of University of Kraków (1919–1920); professor of theoretical physics in University of Warsaw (1920–1953), in the letter of 1–14 September 1912, sent from Kiev, he thanked Natanson for sending the dissertation “Zasady Teoryi Promieniowania” (“The Principles of Theory of Radiation”, published in 1913 – cf. Natanson [1913](#)), which he intended to use at

of professorship at the University of Göttingen in January 1933 (for racial reason, since he was treated as a Jew) and the emigration to England this year, he changed his primary views and became a socialist who was convinced that one had to “fight nationalism in whatever form it appears” (Greenspan 2005, p. 261) “including that of the Jews” (letter from Max Born to Albert Einstein, 22 May 1948 – cf. Born, Max 1936 (*published archival document*)).

⁷⁸ “Shortly before the appearance of our communication of April 1912 [Born, von Kármán 1912] Mr. Debye has, as he told us afterwards, reported his results in the March meeting of the Swiss Physical Society and published a short note in the *Archives de Genève*, March 1912, p. 256 [Debye 1912]. Further, Mr. Natanson claims – based on a communication to the February session of the Cracow Academy [i.e. Natanson 1912b] – that he first stated the idea which lies at the basis of the treatments of Debye and ourselves. It seems to us that the priority for giving a formulation and an approximate solution of the problem belongs to Mr. Dybye by several days” (Born, von Kármán 1913a, p. 15 fn. 1 in the right column; translated by Mehra, Rechenberg [2001](#), p. 143).

In fact, Natanson (1912b) presented his work in Cracow Academy on 8 January 1912 (and not in February). Moreover, Born and von Kármán (1913a) accept that Natanson was the first to state the idea which lies at the basis of the treatments of Debye (9 March, 1912), and Born, von Kármán (April 1912). Therefore, the priority of this idea belonged to Natanson. However, in his biography (cf. Born [1978](#), pp. 141–142), Born neglects this aspect and mentions only Debye's contribution.

Hence, to my current knowledge and in contrast to N. Nagasawa (cf. fn. 37 above), this citation of Born's and von Kármán's (1913a) is the first citation of Natanson's fourth article (i.e. 1912b).

⁷⁹ Regarding the scientific degrees in the Tsarist Russia, cf. Flin, Panko [2015](#).

- the University of Kiev in the spring semester of 1913 during the lectures on theoretical physics of “Theory of radiation”.⁸⁰
- 16) Kazimierz Fajans (1887–1975), a physico-chemist (from 1911 an assistant, from 1913 an associate professor at University of Karlsruhe; from 1917 a professor of University of München) knew and appreciated Natanson’s monograph on radiation and related matters ([1913](#)).⁸¹
 - 17) Emil Lampe (1840–1918) informed about Natanson’s work of [1913](#) in his note in *Jahrbuch über die Fortschritte der Mathematik* ([1914](#)). (However, it was only a bibliographic record of this work and keywords).
 - 18) Stanisław Loria (1883–1955), who was a Ph.D. student of Natanson in 1907, informed about Natanson’s article of [1913](#) in his note in *Jahrbuch über die Fortschritte der Mathematik* ([1914](#)). It is quite probable that he knew all the other Natanson’s works on the subject under discussion, since he corresponded with Natanson and appreciated him as a very good physicist⁸².

⁸⁰ Cf. the letter from Czesław Białobrzewski to Władysław Natanson, Leningrad, 1–14 September 1912 (see Białobrzewski 1912 (*archival document*)). About Białobrzewski’s scientific career cf. Ścisłowski 1954; 1979; Wróblewski [2016](#), pp. 335–339. He is, among other things, a precursor of research on the thermodynamic equilibrium of a star, modeled as a free gas sphere (he did this before Arthur Stanley Eddington’s research) – cf. Białobrzewski [Białobrzewski] 1913 (the work was presented by Natanson on 5 May 1913 at the session of the Academy of Arts and Sciences in Kraków).

⁸¹ Cf. the letter from Kazimierz Fajans to Władysław Natanson, Karlsruhe, 4 November 1913 (see Fajans 1913 (*archival document*), folium 3 recto). About Fajans’s scientific career, cf. Hurwic [1988](#); 2000a; [2000b](#). Fajans was nominated 6 times for a Nobel Prize in Chemistry: in 1928 by [Fritz Arndt](#), [Heinrich Biltz](#), [Walter Herz](#), [Ernst Koenigs](#), [J. Meyer](#), and in 1934 by [Mieczysław Centnerszwer](#).

⁸² In his recollections from his stay in University of Breslau (1907–1908), Max Born makes the following description of Stanisław Loria and formulates the following opinion on Loria’s teacher Ladisław Natanson: “The last in our group, Stanisław Loria, was a Pole from Cracow, hence at that time a subject of the Austro-Hungarian monarchy. [To this group belonged also Max Born (1882–1970), Rudolf Ladenburg (1882–1952) and Fritz Reiche (1883–1969) – M.K.]. But he was a great Polish patriot and hated the Austrians. [...] He was an enthusiastic physicist and a most charming young man, with fine features and perfect manners. We got on very well. And the only point of friction between us was his exaggerated devotion to his professor, Nathanson [sic! – it should be Natanson], in Cracow, whom he declared to be one of the greatest physicists.

- 19) Leopold Infeld (1898–1968), who was the Ph.D. student of Natanson in 1921 and corresponded then with his teacher, knew Natanson’s works of [1913](#)⁸³ and 1924b, i.e. [1924a](#), pp. 125–151, and his other works.⁸⁴

We considered this to be a nationalistic overstatement [sic! – M.K.], and contradicted him; but on the other hand this admiration for his master was rather touching, and we soon ceased our objections” (Born [1978](#), pp. 124–125).

Unfortunately, Born does not explain why he thought that Loria’s admiration for Natanson was “to be a nationalistic overstatement”. To date, however, there are no historical sources that would support Born’s statement.

As a side note: Born learned from Fritz Reiche and Stanislaus Loria about Einstein’s work on special relativity (1905): “[...] When I later (1907–1908) tried to develop my experimental skills at the Institute presided over by Lummer and Pringsheim in my home town of Breslau, I joined an active group of young physicists, including Rudolf Ladenburg, Fritz Reiche and Stanislaus Loria. We studied the more recent physics literature and reported on what we had read. When I mentioned Minkowsky’s contributions to the seminars in Gottingen, which already contained the germ of his four-dimension representation of the electromagnetic field, published in 1907–8, Reiche and Loria told me about Einstein’s paper and suggested that I should study it. This I did, and was immediately deeply impressed. We were all aware that a genius of the first order had emerged. But nobody knew anything about his personality or his life, except that he was a civil servant at the Swiss Patent Office in Berne. Then Ladenburg decided to look him up during a holiday trip, and his account was the first I heard of Einstein the man. Even then he was as he appeared later: completely unpretentious, simple and modest in his habits, kind and friendly, yet witty and humorous. Ladenburg was enthusiastic and made us curious about the great unknown” (Born, Einstein [1971](#), p. 1).

There is another description of this fact given by Leopold Infeld: “My friend Professor Loria told me how his teacher, Professor [August] Witkowski (and a very great teacher he was!), read Einstein’s paper [1905; on special relativity] and exclaimed to Loria: ‘A new Copernicus has been born! Read Einstein’s paper’. Later, when Professor Loria met Max Born at a physics meeting [in fact, it was in 1907–1908 at the University of Breslau, Loria was then a doctor], he told him about Einstein and asked Born if he had read the paper. It turned out that neither Born nor anyone else had heard about Einstein. They went to the library, took from the bookshelves the seventeenth volume of *Annalen der Physik* and started to read Einstein’s article. Immediately Born recognized its greatness and also the necessity for formal generalizations” (Infeld 1950, p. 44; cited by Wróblewski [2014](#), p. 262).

⁸³ Infeld [1958](#), p. 134.

⁸⁴ He wrote a review of the book *Oblicze natury* and mentioned there a paper “On radiation” (=Natanson 1924b). Cf. letter from Leopold Infeld to Wladyslaw Natanson (Warszawa, 9 February 1926) (see Infeld 19126a (*archival document*), folium

- 20) The offprint of Natanson's article of 1912 (1912b) was in possession of the Department of Theoretical Physics, University of Uppsala and it is stamped: "Mek. Sem. Uppsala".⁸⁵
- 21) Otto Halpern (1899–1982), an assistant of professor Hans Thirring (1888–1976) of University in Wien, and Albert Einstein (1899–1982) have known about the existence of Natanson's article (1911c), because Halpern informed Einstein about Krutkov's article (1914a) in the letter of 26 August 1924, which criticized a derivation of Planck's radiation formula with Bose's assumption of independent light quanta in the agreement and on the basis of Ehrenfest [1911a](#) and Krutkov 1914a.⁸⁶
- 22) Witold Jacyna, a Polish physicist from Institute of Metrology, Leningrad, U.S.S.R. and an author of articles in thermodynamics at the *Physikalische Zeitschrift* and the *Physical Review*, knew the second version of Natanson's article "O promieniowaniu" ("On radiation" – Natanson 1924b, i.e. Natanson [1924a](#), pp. 125–151), however, he was not an advocate of quantum ideas.⁸⁷

4–5; Infeld [1926c](#), p. 5 (a review). Moreover he knew the other Natanson works on statistics of radiation and related issues, because in the letter to Wladyslaw Natanson (from Piaseczno, on 26 July 1926) Infeld mentioned as example of great works Natanson's works about radiation (see Infeld 19126b (*archival document*), folium 8 recto and verso).

⁸⁵ Andersson 2018. It is still to buy at the [Antikvariat Thomas Andersson at Uppsala, Sweden](#).

⁸⁶ See Halpern 1924a (*published archival document*). Moreover, Albert Einstein, during his stay in Leyden from 4 to 24 October 1924, discussed this objection with Ehrenfest, which is followed by Einstein's statement from his article: "Mr. Ehrenfest and other colleagues have faulted Bose's theory of radiation and my analogous one for ideal gases for not treating quanta, or molecules, as statistically mutually independent structures, without specifically pointing out this circumstance in our paper" (Einstein 1925a (dated: December 1924; presented: 8 January 1925; published: 9 February 1925) / (reprinted) 2015a, [Doc. 385](#); (English translation) 2015b, [Doc. 385](#) (cf. also its fn. 6).

⁸⁷ Moreover, he thought that all Natanson's monographs known to him ([1908](#); [1924a](#); [1928](#); [1929–1930](#); [1934](#)) were very well written, including especially his monograph entitled *Pierwsze Zasady Mechaniki Undulacyjnej* (*Principes fondamentaux de la Mécanique Ondulatoire*) ([1929–1930](#)), and worth to translate them into foreign languages. He even tried himself to make their translations into Russian, but he did not realize this aim finally. He wanted also to bring to translate into Russian Natanson's monograph entitled

- 23) Leon Lichtenstein (1879–1933), a Polish mathematician, professor of mathematics of Technical University of Charlottenburg, University of Münster, and University of Lipsk, one of the founders and the first editor-in-chief (1918–1933) of the journal *Mathematische Zeitschrift* (founded in 1918), and an editor of *Jahrbuch über die Fortschritte der Mathematik* (1919–1927), appreciated very much Natanson's *Oblicze natury* (1924a; including the second version of Natanson's article "O promieniowaniu" ("On radiation" – Natanson 1924b, i.e. Natanson [1924a](#), pp. 125–151), and also "Zasady teorii promieniowania" (1913) (The Principles of Theory of Radiation).⁸⁸
- 24) Waclaw Dziewulski, a physicist, from October 1919 worked in Stefan Batory University in Wilno, knew the second version of Natanson's article "O promieniowaniu" ("On radiation" – Natanson 1924b, i.e. Natanson [1924a](#), pp. 125–151).⁸⁹
- 25) Walther Gerlach (1889–1979) and Alfred Landé (1888–1976) not only knew Natanson's article (1911c) – cf. Landé 1925 (letter to Natanson, 18 November 1925), but also cited it in their article (Gerlach, Landé 1926, p. 170, fn. 2; reprinted in Landé [1988](#), p. 245, fn. 2).⁹⁰

Hence, it is evidenced that many Polish scientists (physicists, chemists, and mathematicians) appreciated Natanson's works on statistical theory of radiation and related matters. They did so, not only in private

Pierwsze Zasady Mechaniki Undulacyjnej (*Principes fondamentaux de la Mécanique Ondulatoire; 1929–1930*), and "he had a semi-formal pledge as to the favorable resolution of the question, and a very positive response to this request from [I.A.] Krutkov", but "the matter has stopped somehow", cf. the letter from Jacyna to Natanson from Leningrad on 11 January 1936 – see Jacyna 1936 (*archival document*).

⁸⁸ Cf. two letters from Leon Lichtenstein to Wladyslaw Natanson from Lipsk on 29 May 1927, and from Wildungen on 19 August 1928 – see Lichtenstein 1927 (*archival document*); 1928 (*archival document*). On Lichtenstein cf. Przeworska-Rolewicz [1979](#); Gittel [2014](#); *Wikipedia* [2019h](#).

⁸⁹ Cf. letter from Waclaw Dziewulski to Wladyslaw Natanson from Wilno on 29 December 1923, folium 110, where he thanked for sending him *Oblicze Natury* ([1924a](#)).

⁹⁰ To my current knowledge and in contrast to N. Nagasawa (cf. fn. 37 above), it was the fifth citation of this article.

letters, but also in reviews of Natanson's works (cf. Loria 1914, Infeld [1926c](#), p. 5) and in their own scientific articles (cf. Zakrzewski 1911).

From this entire survey (cf. Sections 4.2–4.3) it is also evidenced that Natanson's works on quantum statistics and Natanson himself were known to many scientists of the 1910s, including those who focused their research on the problem of the black-body radiation, the quanta and new (quantum) theories of gases, liquids and solid bodies, especially Zakrzewski (1911) – who was the first to cite the English version of Natanson's article (1911a); Planck (1911; 1912) – who was the first to cite the German version of Natanson's article (1911c); Sommerfeld and Ehrenfest (letter from Ehrenfest to Sommerfeld, October 16, 1911) – who knew the German version of Natanson's article (1911c); Born, von Kármán (1913a) – who was the first to mention Natanson's article (1912b); Krutkow (1914a; 1914b) – who knew both English and German versions of Natanson's article (1911a; 1911c); Landé (letter to Natanson, 18 November 1925); and Gerlach, Landé (1926/reprinted [1988](#)) – who cited the German version of Natanson's article (1911c).⁹¹

In other words, A. Kastler's thesis (cf. Section 4.6) that Natanson's article (1911c) “unfortunately remained unnoticed and unmentioned” (Kastler 1983, p. 617) is wrong since a part of scientists, including eminent and prominent ones, knew this article and valued it highly.⁹²

⁹¹ Hence, the received view that it was only in 1967 that Friedrich Hund as the first scientist appreciated Natanson's works on Planck's theories of radiation and related issues (cf. Section 4.3) is wrong.

⁹² Moreover, since Natanson was used to send his publications to many scientists (it is evidenced by his large scientific correspondence), and this happened also in the case of his correspondence concerning the issues of radiation theory and related issues, it is very probable that his works were known to many other scientists. Since in the case of his two articles published in the *Bulletin International de l'Académie des Sciences de Cracovie, Classe des Sciences mathématiques et naturelles. Série A: Sciences mathématiques / Anzeiger der Akademie der Wissenschaften in Krakau. Mathematisch-Naturwissenschaftliche Klasse. Reihe A: Mathematische Wissenschaften* he had on disposal even 100 copies in each case. This number of copies was mentioned by a Polish physicist, professor of electro-technics and mechanics in the Institute of Engineers of Communication in St Petersburg, and a historian Henryk Merczyng (1860–1916) in his letter from St. Petersburg on 21 August 1912 to Władysław Natanson – see Merczyng 1912 (*archival document*). On H. Merczyng's scientific biography, cf. Zuzga [1987](#).

On the other hand, F. Hund's thesis (cf. Section 4.6), according to which "in 1924 Natanson's arguments had been already forgotten" (Hund 1967, p. 123), is right in the case of Bose and Einstein, since it is true that in 1924–1925 they overlooked subtleties of the issue of indistinguishability of physical objects known to Natanson. However, F. Hund's thesis is wrong in the case of Paul Ehrenfest and their colleagues, including Viktor R. Bursian, Yuri A. Krutkow and Otto Halpern, who did not forget Natanson's considerations, and caused that Einstein was informed on these subtleties of the issue of indistinguishability of physical objects (this issue will be explained in Section 7).

5. Why Natanson's achievements were unnoticed and forgotten by eminent and prominent scientists? The external factors in the reception of the publications⁹³

Four explanations of this fact have been proposed:

A "geographical isolation" explanation by L. Navarro, E. Pérez (2004)

His [Ehrenfest] geographical isolation thus may have contributed to the neglect of his 1911 paper even though it was published in the widely read *Annalen der Physik*. The work of Ioffé, Natanson, and Ishiwara suffered a similar fate. We are not in position, however, to go into this question further in this time (Navarro, Pérez 2004, p. 137).

This interpretation is defective because "the geographical isolation" does not explain the lack of reception of the publication, which appeared in the leading physical journal in a country (Germany) that at the time "was not geographically isolated".

⁹³ The basis of this section is A. Kastler's thesis, that Natanson's article (1911c) "unfortunately remained unnoticed and unmentioned" (Kastler 1983, p. 617), which was accepted by some interpreters of Natanson's achievements. Though the previous section showed that this thesis is wrong, in this section the views of several interpreters of Natanson's achievements who accepted it and explained it in different manners are analysed. The section focuses on the lacks of these explanations.

However, this kind of isolation of German science started only with WWI⁹⁴ and then the foundation of the International Research Council in 1919.⁹⁵ Therefore, L. Navarro's and E. Pérez's term: "geographical isolation" conceals the real lack of openness to the achievements of scientists from such territories as Russia (Ehrenfest and Ioffé worked at the Saint Petersburg Academy), the Austrian governed part of Poland (Natanson from Kraków) and Japan (Ishiwara from Tokyo).⁹⁶

A psychological and sociological explanation by J. Spalek (2006)

Natanson visited Einstein in Berlin 1914 [sic! – M.K.]⁹⁷ and probably did not mention his own work, as he was a shy person [sic! – M.K.] (Spalek 2006).

He worked in an isolated, local scientific environment and published his works in little known journals [sic! – M.K.] (Spalek 2009).

However, having an in-depth knowledge of the biography of Władysław Natanson, it is certain that he was not a shy person!

⁹⁴ During the World War I (from 1914 to 1918) many German scientists took active part in the so-called "Der Krieg der Geister" ("The War with spiritual weapons"). They created, among others, two important documents: the *Aufruf 'An die Kulturwelt!* ("A Call to the Civilized World") of 4 October 1914, signed by ninety-three German scientists (among others, Walther H. Nernst, Wilhelm Röntgen, Wilhelm Wien, Max Planck, and Arnold Sommerfeld), and the *Erklärung der Hochschullehrer des Deutschen Reiches* (Declaration of University Teachers of the German Empire) of 16 October 1914, signed by over four thousands teachers. It caused an isolation of German scientists from the international community. Cf. Wolff 2003 (English)/2007 (German); Somsen 2008; *Wikipedia* 2009; Kleinert 2010; von Ungern-Sternberg 2014.

⁹⁵ I will refer to this issue below in the context of Natanson himself.

⁹⁶ Today's equivalent of this problem is the overrepresentation of English-language journals in Scopus and WoS, and the widespread habit of not quoting publications from "geographically isolated countries". Of course, nowadays, in the age of the Internet, there are no geographical barriers that will limit the dissemination of scientific knowledge. The only barriers are mental barriers existing in human minds. They create scientific ghettos and world temples of knowledge.

⁹⁷ Natanson with his family went for a summer holiday 1914 to Belgium (Westende, near Ostende). He planned to stay there from August to the first half of September.

Moreover, his works could be understood by a broad international community of physicists, because he published his major works in English, French and German (and also in Polish). He was known well by this international community, because he was a member of the Physical Society of London (from 1886), a member of the Deutscher Physikalischen Gesellschaft (from 1913)⁹⁸, the Council of the Société Française de Physique (from 1914), the Polish delegate to the General Assembly of the International Research Council (from 1919), a co-founder of the International Union of Pure and Applied Physics (1922) as a representative of Poland and Vicepresident of this union (1925–1931). He corresponded with many leading scholars of his times such as Max Planck, Albert Einstein, Hendrik Antoon Lorentz, Arnold Sommerfeld, Maria Skłodowska-Curie, etc.⁹⁹

The outbreak of WWI forced him to stay in this country, and then from 24 November 1914 until the first days of September (at least 3rd September) 1915 – they spent in Berlin (letters on 29 June 1914, 17 November 1914, and 11 September 1915 from Władysław Natanson to Marian Smoluchowski – see Natanson 1914a (*archival document*), folium 151 verso; 1914b (*archival document*), folium 153 recto; 1915 (*archival document*), folium 176 verso).

During this stay Einstein met Natanson personally (and his family) and became friends (cf. Średniawa 1996, pp. 76–77; 2006, p. 260). Their first meeting was at the beginning of January 1915. Einstein informed his friend Heinrich Zangger about this event on 11 January 1915: “In recent days I made the acquaintance of our colleague Natanson from Cracow, a fine theoretical mind. [...] He is a Polish Jew and grew up in Russia, now 50 years of age. I quickly took a liking to him as I rarely do with people; blood runs thicker than water!” (Einstein 1998a, Doc. 45a, p. 28 / English translation: Einstein 1998b, Doc. 45a, pp. 15–16; see also Konieczny 2011, p. 255).

The last meetings were before 4 September 1915. Natanson informed his friend Smoluchowski about these events: “In Berlin, before leaving, [I had] yet a few more nice and beautiful conversations with prof. Einstein. He promised to come to us to Krakow” („W Berlinie z prof. Einsteinem jeszcze kilka nader miłych i ślicznych rozmów, przed wyjazdem. Obiecał, że przyjedzie do nas do Krakowa” (Letter on 11 September 1915 from Władysław Natanson to Marian Smoluchowski – see Natanson 1915 (*archival document*), folium 177 verso).

⁹⁸ Cf. Planck 1913 (*archival document*) – see below, Appendix 3 for its transcription and translation into English.

⁹⁹ Cf. Natanson 1933/1958; Biblioteka Jagiellońska, Natanson’s correspondence; Konieczny 2011, p. 255.

For example: he was well known by Hendrik Antoon Lorentz, who – in the letter of 20 August 1914 to Pieter Zeeman – wrote:

From foreign friends and acquaintances I hear hardly anything. Incidentally I heard that Stark was inducted in the Feldsturm, and that Natanson is in Westende near Ostende with wife and children and does not know how and when he will be coming home again (Lorentz 1914 (*published archival document*), in: Lorentz 2018, Doc. 160, p. 416; English translation: p. 417).

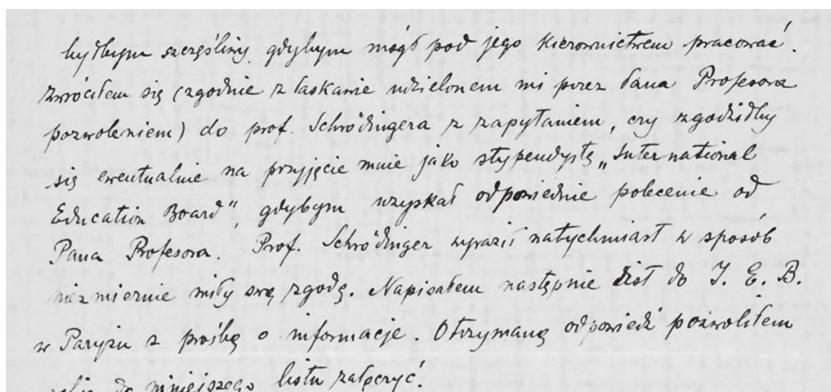


Fig. 15. The excerpt of Infeld's letter sent to Natanson from Warszawa, dated 5 October 1928. Source: Infeld 1928 (*archival document*), folium 11 verso. Photo: © Michał Kokowski.

Moreover, he was appreciated by Erwin Schrödinger as evidenced by letter from Leopold Infeld to Władysław Natanson (Warszawa, 5 October 1928):

Zwróciłem się (zgodnie z łaskawie udzielonem mi przez Pana Profesora pozwoleniem) do profesora Schrödingera z zapytaniem, czy zgodziłby się ewentualnie na przyjęcie mnie jako stypendysty „International Education Board”, gdybym uzyskał odpowiednie polecenie od Pana Profesora. Prof. Schrödinger wyraził natychmiast w sposób niezmiernie miły swą zgodę (Infeld 1928 (*archival document*), folium 11 verso).

I have turned to Professor Schrödinger (according to the permission granted me by you, dear Professor) asking

if he would agree to take me as a holder of the “International Education Board” scholarship if I received the appropriate instruction from you, dear Professor. Professor Schrödinger was exceptionally kind and immediately gave his me consent [translation – M.K.].

Hence, there is no doubt that though Natanson lived in Poland and was active in a little Polish local scientific environment, he was well known in the international community of physicists.¹⁰⁰

Moreover, it appears that there is another explanation of the enigmatic behavior of Natanson regarding Einstein. Some light is shed on this by a similar event that took place in 1931, fourteen years after the death of Marian Smoluchowski. In the letter of 10 July 1931 to Tadeusz Smoluchowski (the brother of Marian Smoluchowski), Natanson commented on E. Arendt’s remarks (1931) on Natanson’s work (1888a/[1888b](#)) related to the later analysis of the problem of Brownian motions by Smoluchowski in 1900–1910s and stated:

[Marian Smoluchowski] did not know about this work; I wanted to tell him about it a few times, but I always postponed it in order not to allow him to think that I wanted to revindicate a part of his discoveries for myself. I had so much devotion and attachment for him, the most cordial, almost fraternal, that I did not want to put a question *de priorité* between us [...] [translation – M.K.].¹⁰¹

We will understand these wordings even better when we add a statement here from a letter of 15 September 1933 from Arkadiusz Piekara (then in Paris) to Władysław Natanson:

¹⁰⁰ I develop in this paragraph the thought firstly expressed by Matthew Konieczny (2012, p. 74/75). In 2010/2011, I was his scientific supervisor. He stayed then in Kraków as a scholarship holder of the Fulbright Foundation (he got the Fulbright Junior Research Award).

¹⁰¹ “[Marian] nie wiedział o tej pracy; parę razy chciałem Mu o niej powiedzieć, ale odkładałem zawsze, żeby nie nasuwać Mu myśli, jakoby miał chęć odzyskać Jego odkrycie rewindykować dla siebie. Miałem dla Niego tyle czci i przywiązania tyle, najgorętszego, prawie że braterskiego, iż nie chciałem kwestyj *de priorité* pomiędzy nami [...]” (cited by Teske 1970a, p. 143).

I am reminded of the words of the Venerable Rector [i.e. Natanson], spoken a year ago in Warsaw. “Science is not our life’s goal, it is only its color” [translation – M.K.].¹⁰²

Hence, Władysław Natanson was not a shy person, but a very friendly and polite one, and physics was not so important to him to risk weakening his friendship with Marian Smoluchowski.

Therefore, perhaps, it was for the same reason that Natanson did not mention his works of 1911–1913 during his talks with Einstein in Berlin in 1915. Nevertheless, from my own perspective – I agree in this point with Józef Spalek – such interpretation seems unlikely, since in 1915 these subjects were still very interesting and “hot” to discuss with Albert Einstein. However, this “unlikeliness” is only my guess not based on any historical source, so it can be wrong.

Moreover it is not true that Natanson worked in an *isolated*, local scientific environment and published his works in little known journals. First of all, this local scientific environment was not isolated from international science as evidenced by the same level of scientific works of, for instance, Smoluchowski, Olszewski and Natanson. Then, Natanson’s articles appeared, among others, in *Wiedemann’s Annales* (1885/1886), *Zeitschrift für Physikalische Chemie* (1892; 1894; 1895; 1896; 1902; 1903), *Comptes Rendus de l’Académie Française des Sciences* (1893), *Philosophical Magazine* (1895; 1901; 1919, 1933), *Annalen der Naturphilosophie* (1901), *Journal de Physique Théorique et Appliquée* (1903), *Journal of Physical Chemistry* (1903), *Journal de Physique* (1909), *Physikalische Zeitschrift* (1911c), and also *Bulletin International de l’Académie des Sciences de Cracovie, Classe des Sciences mathématiques et naturelles. Série A: Sciences mathématiques / Anzeiger der Akademie der Wissenschaften in Krakau. Mathematisch-Naturwissenschaftliche Klasse. Reihe A: Mathematische Wissenschaften* (1893; 1895; 1897; 1898; 1899; 1901; 1902; 1911a; 1912b).¹⁰³ The latter journal published works in German, French, English or Polish (so it promoted international communication in then science), and these works were mentioned and reviewed by the *Science Abstracts* and the *Jahrbuch über*

¹⁰² “Przypominają mi się słowa Czci. Pana Rektora, wypowiedziane przed rokiem w Warszawie. Nauka nie jest celem życia, jest tylko jej barwą” (Piekara, Arkadiusz 1933 (*archival document*), folium 257 verso).

¹⁰³ Cf. the list of Natanson’s publications Natanson [1879–1937](#) (pp. 300–307).

die Fortschritte der Mathematik (contemporary analogies of present-day Scopus and Clarivate databases).

Hence, it is not legitimate to claim that non-Polish journals mentioned above – including e.g. the *Physikalische Zeitschrift*, a leading journal on an international scale – or the *Bulletin International de l'Académie des Sciences de Cracovie... Série A...* were isolated in any sense from international audience. Since in the case of the reception of Natanson's scientific publications, neither the language nor the place where he published his works were barriers, they had to be mental and political barriers existing in the minds of scientists.

An epistemological explanation by Konieczny (2010)

Despite advanced work on the very issues at stake at Solvay in 1911, and extensive contact with Western European institutions and practitioners of physics, why were no scientists working at universities in the peripheries of East-Central European empires invited? Ultimately, I argue that their exclusion was not based on an ignorance of their work or a prejudice against scientific workers on the periphery per se, but rather that Eastern European physicists approached the issues at stake at Solvay from fundamentally different epistemological and ontological dispositions, which rendered their work beyond the bounds of the dominant scientific discourse and thus intellectually incompatible (Konieczny 2010).

I think that this thesis is too bold. It is better to say in this context that the nationalist ideology that spread in 1910s Europe made a free flow of information and a free reception of ideas very difficult or even impossible. Natanson was not a nationalist but a Renaissance person. He was not only a physicist, but also a historian and a philosopher of science, and a talented humanist, a philosophical literary critic, the author of literary portraits of world-famous writers (Shakespeare, Shelley, Dostoyevsky, ...), a biographer of outstanding physicists (Newton, Faraday, Maxwell, Smoluchowski, ...),¹⁰⁴ who received the gold medal of

¹⁰⁴ Cf. Natanson 1908; 1924a; 1928; 1934; 1937; 1977.

“Wawrzyn Akademicki” (Academic Laurel) of the Polish Academy of Literature in 1936 “for outstanding critical-literary, scientific and journalistic work in the field of belles-lettres”.¹⁰⁵ And being such a creative man, he did not fit the nationalistic *Weltanschauung* or *Zeitgeist* that dominated the Europe of these times.

A political explanation by M.J. Konieczny (2011; 2012)

The so-called “Western science” at the turn of the 19th and 20th centuries was limited by the nationalistic approach, especially with regard to French and German science.¹⁰⁶ In contrast, Władysław Natanson was a true European scientist, that is, being free of nationalism, he appreciated the value of works of authors irrespective of their nation or state of origin; his works on quantum statistics were not accepted, since he did not belong to any “scientific party”.

I agree with this interpretation partially: it is true that all his education and scientific activities, including publications, prove that he drew on the achievements of various linguistic cultures (especially British, French, German, and American) and he did not belong to any “scientific party” nor to any “nationalist scientific camp” (British, French, German or American).¹⁰⁷ In support of this thesis, I will quote Natanson’s autobiographical statement of 1933:

Throughout my life, I have tried with all my strength to learn from the most esteemed masters, even if they had not been alive for a long time. How much have I learned from Newton, from Lagrange, Kelvin, Clausius, from J.W. Gibbs, from G.G. Stokes, from Lord Rayleigh, P. Duhem, and H.A. Lorentz. My first beloved role-model and commander has always been J. Clerk-Maxwell. Communing with the works of genius creators leaves in the mind

¹⁰⁵ Cf. “Eine” [2007](#); Kokowski 2009; *Wikipedia* [2019g](#), and first of all his brilliant essays written in Polish – Natanson [1924](#); [1928](#); [1934](#); [1937](#).

¹⁰⁶ It is well known problem – cf. also Kleinert [1978](#); Wolff, Stefan L. [2003](#) (English)/[2007](#) (German); Somsen [2008](#); Eckert 2013a/2013b; Gordin [2015](#); Fox 2016.

¹⁰⁷ The existence of “national scientific camps” in science causes the existence of serious incoherency between different national histories of science. Cf. Wróblewski ([2006](#)) regarding incoherency of national histories of science, and Kokowski (1993; 1994; [1997](#); 2009; 2010) regarding Natanson’s views.

and soul traces and effects which, in my opinion, no lectures – be it seminars or foreign ones – can bring about [translation – M.K.]¹⁰⁸.

On the other hand, I think that M.J. Konieczny did not provide solid empirical evidence – i.e. explicit statements in archival or printed correspondence or articles and books of the scientists of his epoch – that Natanson’s “works on quantum statistics were not accepted, since he did not belong to any «scientific party»” (but see below).

A political explanations by N. Nagasawa (“Minamida” 2009; Nagasawa 2018)

According to N. Nagasawa (“Minamida” 2009; Nagasawa 2018, pp. 404–407), one of the reasons, why Natanson’s paper was forgotten in 1924 (when Bose’s article appeared) within the physics community was a nationalist climate guarding the European physics community, and especially the German one (he referred in this point to S.L. Wolff’s work (2003)).

An expression of this attitude was to be Arnold Sommerfeld’s letter dated 1 November, 1919 to Adalbert (Wojciech) Rubinowicz (a Polish scientist), a former assistant of Sommerfeld. Rubinowicz was asking for Sommerfeld’s help to look for the academic position. Sommerfeld advised him:

It would be very difficult to find positions in Poland. Natanson is not reliable because his activity is limited within his domestic community [sic! – M.K.]. You should contact M. [Maria – M.K.] Curie. She is not poisoned by chauvinism (Nagasawa 2018, p. 407; quoted also in “Minamida” 2009).

However, still according to N. Nagasawa, being familiar with the biography of Władysław Natanson (cf. Klecki 1938), we should not think that he was a chauvinist (Nagasawa 2018, p. 406). Therefore, Sommerfeld

¹⁰⁸ “Przez całe życie starałem się ze wszystkich sił uczyć od najwyższych Mistrzów, chociażby już Ich dawno nie było na ziemi. Ileż nauczyłem się od Newtona, od Lagrange’a, Kelwina, Clausiusa, od J.W. Gibbsa, od G.G. Stokesa, od Lorda Rayleigh, P. Duhema, H.A. Lorentza. Najpierwszym, miłowanym wzorem i wodzem był zawsze J. Clerk-Maxwell. Obcowanie z dziełami genialnych twórców pozostawia w umyśle i w duszy ślady i skutki, których mym zdaniem, żadne wykłady – seminaryjne, zagraniczne – wydać nie mogą” (Natanson 1933/1958, p. 115).

– who was immersed in the toxic climate of European chauvinisms (German, English, French, etc.) and had a nationalist bias against Natanson – was probably not eager to inform Bose or Einstein about Natanson’s work of 1911 (Nagasawa [2018](#), p. 407).

I have the following comments to N. Nagasawa’s interpretation.

Firstly, if the sentences cited above by Nagasawa are a faithful translation of Sommerfeld’s sentences, the conjunction of these sentences allows two interpretations: the first – weaker – that Natanson is not reliable or even the second – stronger – that he was not reliable since he was a chauvinist.¹⁰⁹

Secondly, I can indicate solid arguments in favour of Nagasawa’s guess that Natanson was not a chauvinist (see below).

And finally, I do not agree that Arnold Sommerfeld wrote the quoted above statements. These are only Nagasawa’s guesses of the real essence of these statements, expressed in two letters from Sommerfeld to Rubinowicz (Stockholm, 1 October 1919; München, 1 November 1919), which is not easy to determine (it is caused by the very style of Sommerfeld’s handwriting). Upon reading these two Sommerfeld’s letters (see Appendix 4), on the one hand, it is not clear if Sommerfeld thought that Natanson was a chauvinist, and, on the other hand, it is clear that though Sommerfeld did not know “how strong the chauvinism of the Poles was”, nevertheless, in his own opinion, perhaps the Poles, like the French, “were crazy” (in their chauvinism), and perhaps Marie Curie’s stance was not entirely chauvinistic.¹¹⁰

¹⁰⁹ Regarding the latter interpretation: It follows from the second quoted sentence that Natanson was not reliable since his activities were limited only to the domestic Polish community; and the second and the fourth sentences – that in contrast to Natanson – M. [Marie] Curie (i.e. Maria Skłodowska-Curie) was not a chauvinist. Hence, Natanson was a chauvinist.

¹¹⁰ As it is well known from the biography of Arnold Sommerfeld, he had racist or nationalistic, or chauvinistic episodes in his life. E.g., in 1907 in a private letter to Einstein, Sommerfeld stated that perhaps in the theory of relativity “the abstract-conceptual nature of the Semite” was expressed (however, he accepted later the theory of relativity and was not a supporter of the *Deutsche Physik* or *Arische Physik*) – cf. Kleinert [1985](#), p. 1985; Jansen [2009](#), p. 148. Then, in 1914 he was one of ninety-three signatories of the *Aufruf ‘An die Kulturwelt!* (“A Call to the Civilized World”) of 4 October 1914 (but he wanted this document to be known only to the Germans) – see Wolff [2003](#) (English) / [2007](#) (German). Moreover, when in 1917 he tried to get a job at the University of Vienna, two scholars were selected to take up the professorship

However, irrespective of these Sommerfeld's views, it is certain that Natanson cannot be characterized in this way. This is proved by his entire activities in the Polish and international scientific communities,¹¹¹ by his life-long intellectual fascinations with the Jewish, British, Russian, French, Italian, old Islamic (Sufism) and Polish cultures, by his voluminous correspondence, both scientific (with many famous scientists) and private,¹¹² and finally by the recollections of his friends, participants of the famous intellectual inter- and multi-disciplinary "Symposium" (from Greek: *σμπόσιον*, *symposion*) organized by Natanson in his home for many years.¹¹³ To support these statements, it is worth mentioning Natanson's own words from only three documents of 1918, 1919 and 1933.

The first document is a draft of Natanson's talk (in English) of 1918 from his welcome of the past president of USA Thomas Woodrow Wilson, for the award ceremony of a doctor honoris causa of philosophy in the Jagiellonian University.

post: Sommerfeld himself and Smoluchowski. On 14 March, 1917, a mathematician Wilhelm Wirtinger, the then dean of the Philosophy Department and chairman of the Commission, and his colleague from Felix Klein seminary at Göttingen, tried to make Sommerfeld the only candidate for this position: "Wirtinger had entered a petition «to put you [Sommerfeld] unico loco on the list; Smoluchowski on the other hand, not at all.» The majority of the committee had accepted this petition, although in a minority vote the physicists had insisted on Smoluchowski. Just why Wirtinger, along with the majority of his colleagues, was against Smoluchowski, he explained by writing, «that Sm. [Smoluchowski] is a Pole, and declares himself as such. I can imagine that in the German Reich there is no true appreciation of what this means to us [in the Austro-Hungarian Empire]. For you can always count on your government's being German, whereas for us «Germanness» constitutes a chip the government bargains with in various difficult circumstances to accommodate the other nationalities, so that we have to protect it ourselves.»" (Eckert 2013b, pp. 215–216).

¹¹¹ He was Chairman of the Faculty of Mathematics and Natural Sciences at the Polish Academy of Arts and Sciences, the first President of the Polish Physical Society (1920–1923), Rector of the Jagiellonian University in 1922/1923; a co-founder of the International Union of Pure and Applied Physics in 1922 (as a representative of Poland) and its Vice-president (1925–1931), etc. – regarding the latter issue, see Ossipyan, Yamaguchi 1992.

¹¹² Cf. Natanson's correspondence at the Library of the Jagiellonian University, and at the Archive of the Jagiellonian University.

¹¹³ Cf. e.g. Michalski 1937a (pp. 308–316; in French); 1937b (in Polish); Klecki 1938, p. 32; Kokowski 2009.

I am very glad that it is my duty to welcome you, Mr President, in the name of the our old university. It has often been said that Science is of no nationality; Science indeed is international and gives light to Humanity. Let us remember, however, that Science is also the highest product of national character and national virtue. Let us remember, that Science paves the way for a better understanding of a men and nations. [...] What is now spent on armament will in the future be devoted to the advancement of Knowledge; nations will not seek supremacy over nations, they will seek supremacy over Nature (Draft of speech 1918, Archives of the Jagiellonian University, Spuścizna 10/24).¹¹⁴

We can label Natanson's political stance regarding science as *Olympic internationalism* (in Geert J. Somsen's terminology).¹¹⁵ Hence, Natanson was not a chauvinist. This is confirmed additionally by the *Interim Report on the course of the Constitutional Assembly of the International Research Council held at Brussels, from 18 to 28 July 1919* (*Sprawozdanie tymczasowe z przebiegu Zgromadzenia Konstytucyjnego Rady Międzynarodowej Badań Naukowych odbytego w Brukselli, w dniach od 18. do 28. lipca 1919 roku*) that Natanson (1919) prepared as the only Polish representative at this meeting.¹¹⁶

To a peaceful and unbiased observer it is clear that as a result of the war the scientific world will remain for a long time divided into two camps, separated from each other and mutually averse. Whether we approve of it, or whether we condemn it, whether we consider it justified or

¹¹⁴ Cited by Koniczny 2008, pp. 127–128, who thought mistakenly that it was a welcome in 1924 of scientists from Europe and USA.

¹¹⁵ Regarding the meaning of this term see Somsen 2008, pp. 365–367.

¹¹⁶ To understand well Natanson's comments on Constitutional Assembly of the International Research Council one should add that the aim of the foundation of this institution was the reconstruction of international scientific arrangements after the WWI, but without Germany. During the meeting in Brussels, the representatives of western countries (including President of the Assembly, Professor Alfred Lacroix (France), Minister Alphonse Harmignie (Belgium), and Professor Auguste Gravis, Director of the Class of Sciences of the Belgian Academy) spoke openly for the boycott of German science. This was accompanied by overt hostility towards the domination of the German language in science and the influence of German researchers on the development of science (cf. also Onghena 2011, pp. 283–284; Cock 1983).

incomprehensible, we are dealing here with a mighty and massive phenomenon that we will have to reckon with for years to come. We are connected with the West with our strivings and feelings, our history and culture, our views and needs; we can no longer choose a camp where we want to be. But if we should belong to the Western civilization, then the consequences resulting from it are serious and impose great obligations on us. From among the comments that arise here, I will cite only some. [...] The tendency of the Assembly, many times expressed in public, was “to be separated from the Germans; in all areas of Knowledge, do without the Germans.”¹¹⁷

Referring to this, Natanson pointed out:

Such resolutions, feelings and intentions are not isolated. I inform about them to Akademia [Academy of Arts and Sciences in Kraków], but I do not judge them. I do not make a judgment here, I quote the facts.¹¹⁸

Continuing this thought, Natanson noted that if the authorities of the (Polish) Academy of Arts and Sciences in Kraków and the Polish government adopted such a policy, the number of publications in English and French should be increased, because there is a demand for it in countries belonging to the International Research Council, and this also

¹¹⁷ “Dla spokojnego i nieuprzedzonego obserwatora jest rzeczą widoczną, że świat naukowy, w następstwie wojny, będzie przez długi czas podzielony na dwa obozy, odgradzone od siebie i niechętnie sobie. Czy je pochwalamy, czy też ganimy, czy uważamy je za uzasadnione lub niezrozumiałe, mamy tu do czynienia z potężnym, z tłumem zjawiskiem, z którym przez lata będziemy musieli się liczyć. Jesteśmy związani z Zachodem dążeniami i uczuciami, historią i kulturą, poglądami i potrzebami; nie możemy już dzisiaj wybierać obozu, w którym pragniemy się znaleźć. Lecz skoro powinniśmy należeć do cywilizacji zachodniej, przeto następstwa stąd wynikające są poważne i nakładają na nas wielkie zobowiązania. Z pomiędzy nasuwających się tu uwag przytoczę tylko niektóre [...]. Tendencją Zgromadzenia, wielokroć razy wyrażoną publicznie, było «odgradzić się od Niemców; we wszystkich zakresach Wiedzy obejść się bez Niemców»” (Natanson 1919, p. 26).

¹¹⁸ “Uchwały, uczucia i zamiary podobne nie są odosobnione. Donoszę o nich Akademii, lecz ich nie oceniam. Nie wypowiadam tutaj sądu, przytaczam fakta” (Natanson 1919, p. 27).

should be applied to the journal *Bulletin International*, issued by the Academy of Arts and Sciences, in which the German language prevails.¹¹⁹

Consequently, Natanson's statements above prove again that he was not a chauvinist. This is confirmed also by his autobiography of 1933:

I have never belonged to political parties; I do not belong to any party today either. I was averse and reluctant to political fights. I always wanted to do something, create as much as I could, leave something after myself; fights, polemics disgusted me. In my opinion, what is weak and poor will fall, perish, vanish. Creativity is the best criticism there is.¹²⁰

Finally, it is worth recalling here that during Natanson's stay in Berlin in 1915, Albert Einstein – who for his pacifistic convictions was boycotted during WWI by the nationalistically oriented scientific community of German scientists¹²¹ – found in Władysław Natanson a non-nationalistic, non-chauvinistic soulmate.¹²²

¹¹⁹ Natanson 1919, pp. 27–28. It should be noted that the full name of this journal was: *Bulletin International de l'Académie des Sciences de Cracovie, Classe des Sciences mathématiques et naturelles. Série A: Sciences mathématiques / Anzeiger der Akademie der Wissenschaften in Krakau. Mathematisch-Naturwissenschaftliche Klasse. Reihe A: Mathematische Wissenschaften*. Hence, the journal of the Academy of Arts and Sciences in Kraków (after 1919, the Polish Academy of Arts and Sciences) was open to two then antagonistic camps of French science and German one.

¹²⁰ “Do stronnictw politycznych nigdy nie należałem; nie należą też dziś do żadnego stronnictwa. Walkom politycznym czułem się obcy, niechętny. Pragnąłem zawsze coś czynić, w miarę sił tworzyć, po sobie zostawić; walki, polemiki, budziły we mnie niesmak. Co jest słabe i liche, moim zdaniem, samo upada, zginie, przepadnie. Najlepszą krytyką jest twórczość” (Natanson 1933/1958, p. 119).

¹²¹ “Einstein was one of only four signatories of the pacifist declaration «Manifesto to the Europeans» (“Aufruf an die Europäer”), written by Georg F. Nicolai (1874–1964), [e]xtraordinary [p]rofessor of [m]edicine and [p]hysiology at the University of Berlin, in response to the manifesto «To the Civilized World» (“An die Kulturwelt”)” (Einstein 1915a (*published archival document*). [In:] Einstein 1998a, Doc. 45a, p. 29, fn. 7. Cf. also Wolff 2003, pp. 343–344 (English) / 2007, p. 46 (German).

¹²² Cf. the following excerpts of two letters of 29 December 1915 and 14 September 1917 from Albert Einstein to Władysław Natanson.

“Solange Sie da waren, sind Sie mir der liebste Berliner gewesen; der gemütliche Verkehr mit Ihnen geht mir jetzt sehr ab.” (“As long as you were here, you had been my favorite Berliner; now I miss our relaxed relations very much”) (Einstein 1998a, Doc. 175, p. 231; English translation: Einstein 1998b, Doc. 175, p. 169).

Considering the above comments I agree with N. Nagasawa, that it cannot be ruled out that Arnold Sommerfeld was not eager to promote Natanson's work in 1924 for political prejudices.

6. Who discovered Bose statistics and Bose-Einstein statistics?

As it is already stated in Section 3.4: a) F. Hund (1967) formulated thesis that *Natanson was the first who formulated Bose statistics of "light quanta"*, b) this thesis was accepted by some later commentators of Natanson's works (A. Hermann, A. Kastler, B. Średniawa), A. Bach, Boya, Hentschel, S. Varró, B. Lange, and "Roh Minamida" / N. Nagasawa, c) it is B. Lange's thesis that we should talk about Natanson's statistics (1997a) or, according to P. Mittelstaedt (2013), about "Natanson statistics", d) it is J. Spalek's thesis that we should talk about "Natanson-Bose-Einstein statistics" (2005), and e) it is M. Waniek's and K. Hentschel's thesis that we should talk about "Planck-Natanson-Bose-Einstein statistics" (2011).

Contrary to the above-mentioned scholars I think that – without any exaggeration – W. Natanson or M. Planck may be considered a precursor¹²³ of Bose statistics and Bose-Einstein statistics, but the idea that each of them was the author or a co-author of these statistics is too far-reaching. Regarding W. Natanson, I share the opinion of such researchers as L. Infeld, and S. Bergia with regards to this issue.

The former (Infeld 1958, p. 136; 1964b, pp. 35–36) noted about Natanson that:

„Man fühlt sich immer fremder in dieser harten Welt. Aber man freut sich der wohlwollenden Gesinnungs brüder, wenn sie auch ferne in Krakau sitzen! Hoffentlich führt uns das Schicksal bald einmal wieder zusammen.“ (“One feels increasingly alienated at this hard world. But we appreciate our good-willed brothers of like mind, even if they are located far away in Cracow! I hope fate soon brings us together once again”) (Einstein 1998a, Doc. 380, p. 514; English translation: Einstein 1998b, Doc. 380, p. 373).

Five letters from Einstein are preserved in the correspondence of Natanson kept in the Jagiellonian Library. Regarding these letters cf. Średniawa 1996, pp. 76–77; 2006, p. 260.

¹²³ „Precursors serve a social function in the construction of the collective memory of a community by creating links that provide continuity in time and between successive generations of scientists” (Gingras 2007, p. 371). Cf. also entire section entitled “The Social Function of ‘Precursors’ in Collective Memory” (pp. 371–372).

he was close, remarkably close to the great scientific discoveries, such as the formulation of Bose statistics.

And according to the latter (Bergia 1987, p. 234; reprinted in: Bergia 2009, p. 344):

It must be stressed that Natanson does not suggest a physical interpretation in terms of objects: photons did not exist in 1911.¹²⁴ Still, he is certainly to be considered as a forerunner of Bose statistics.

On the other hand, I cannot agree with the bold thesis stated by A. Bach (1988; 1990; 2008):

That what we now call Bose-Einstein statistics actually had been introduced by Boltzmann in 1877 in the context of establishing the entropy–probability relationship (Bach 1990, p. 2).

BE [Bose-Einstein] statistics was introduced by Boltzmann in the period 1868–1877 as a discrete scheme to derive the exponential distribution (Boltzmann distribution) and to establish the entropy–probability relationship (Bach 2008, p. 3).

The reason is quite simple, since L. Boltzmann and later M. Planck and W. Natanson were not authors of the quantum density of states discovered by Bose in 1924,¹²⁵ and Boltzmann’s combinatoric approach is based on an imaginary fiction not realized in the real world of mechanical objects.¹²⁶

However, I do agree with the advocates of the second and third theses mentioned in Section 3.5 that Natanson was the first to understand the statistical foundations of Planck’s law of black-body radiation, and that Natanson and Ehrenfest were the first to understand the concept identity of physical objects.

¹²⁴ There existed, of course, since Einstein’s paper of 1905, the light quantum, but most physicists (see, for instance, Klein 1970a) did not give much credit to the idea.

¹²⁵ However before S.N. Bose’s article (1924a), several authors already published articles contributed to explain the issue of the phase space elementary cell (h^3). M. Planck was also among them – cf. Ishiwara (1911), Sackur (1911; 1912a; 1912b); Tetrode (1912a; 1912b), Planck (1916), L. de Broglie (1923; 1924). Moreover see: Quarati, Lissia 2013; Abiko 2015; Ebeling; Pöschel 2019, p. 7.

¹²⁶ Cf. fn. 26.

Taking all these into account, I can now answer the question: “Who discovered the so-called Bose-Einstein statistics?”

From the perspective determined by using of the integrated historiographic approach described in the introduction (cf. Section 1), there is no simple answer to such a simple-sounding but very difficult question. The final answer depends on the specific context in which we should formulate it. If we are not interested in the history of physics, our answer is quite simple and clear – they did it, Bose and Einstein in 1924–1925, and nobody else. However, if we take history seriously, our answer should be different. I outline its idea below.

L. Boltzmann formulated “Bose-Einstein statistics” in 1868–1877 (see Bach 2008, p. 3), but it was based on a mathematical trick of “*quant*” of *speed* or *vis viva* (twice kinetic energy) without a physical sense. However, this statistics acquired physical meaning only in a limit case, when the number of quanta of *vis viva* tends to infinity, and the magnitude of *vis viva* tends to zero.¹²⁷ This limit case is named the Maxwell-Boltzmann distribution, because Maxwell found it before Boltzmann in an independent way. Then the Wien-Jeans law of radiation and Planck’s law of black-body radiation were discovered. Planck gave a theoretical explanation of black-body radiation. However, his explanation was not perfect, what was criticized by many authors. Then, Natanson (1911a; 1919c), following in the footsteps of earlier scholars (starting of Boltzmann), understood the statistical assumptions of “Bose statistics” as early as December 2010 (cf. his letter to Smoluchowski of 22 December 1910) – 6 March 1911 (date of presentation of his paper at the meeting of the Academy of Arts and Sciences in Kraków), but he did not give an appropriate physical model of the problem being considered and did not give a quantum derivation of the formula for the density of states what are the merits of Bose.

Einstein, in his earlier papers on the quantum theory of ideal gases, between 1916 and 1924, and Bose in his first paper in 1924, overlooked that the quanta, respectively molecules, should be treated as statistically independent entities. This was noticed by Paul Ehrenfest and “other colleagues” such as Viktor R. Bursian, Iurii A. Krutkov and Otto Halpern (see Cannals, Sauer 2010a, pp. 10, 13, 14) which was indicated by Einstein himself in 1925 in his second paper on Bose-Einstein statistics (Einstein 1925a/2015a, [Doc. 385](#); (Eng. transl.) 2015b, [Doc. 385](#)).

¹²⁷ Cf. Bach 1988; 1990; 2008; Nauenberg 2016, pp. 717–718.

Moreover, Einstein, P. Ehrenfest and “other colleagues” did not realize that the key to understand a difference between particles obeying quantum and classical statistics is not the indistinguishability of particles but the single-particle states: quantum states are discrete and classical states are dense.¹²⁸ However, the problem was well understood by Natanson as early as December 1910 – 6 March 1911, and “scholar colleagues” around the world knew his works on radiation and specific heat (1911a; 1911c; 1912b), because he sent these works to many of them (they had to know at least his work on radiation published in the *Physikalische Zeitschrift* (1911c), the leading journal of those days).

There is also no doubt that Boltzmann and Planck had great merits in formulating the new quantum statistics of radiation, though they didn't discover it.

Therefore, to emphasize the key achievements of Boltzmann, Planck, Natanson, Bose and Einstein, it is worth introducing a new convention and talking about *Boltzmann-Planck-Natanson statistics* (for the old radiation theory) and using the terms *Bose statistics* (for the new quantum theory of radiation) and *Bose-Einstein statistics* (for radiation and matter, i.e. for particles with a spin of 0, 1, 2, ...), taking into account the improvements introduced by Natanson, Ehrenfest, ...). It is also useful to introduce another convention and talk about *Boltzmann-Planck-Natanson-Bose-Einstein statistics* – in this case we do not pay much attention to the physical mechanisms or postulated quasi-entities of theories and we generalize the terms “Planck-Bose statistics”, introduced by John Hendry (1984); “Natanson-Bose-Einstein statistics”, introduced by Józef Spalek (2005), and “Planck-Natanson-Bose-Einstein statistics”, introduced by Magdalena Waniek and Klaus Hentschel (2011). Nevertheless we should remember that such terms are only conventions, which conceal and omit a number of additional co-authors.

¹²⁸ Cf. Winterbon 1988, p. 334 (to be clear, Winterbon considered the problem of understanding the differences between the quantum and classical statistics and the indistinguishability of particles, and he did not evaluate the views of the scientists mentioned by me); Saunders 2006, p. 17.

7. Which historian of physics was the first to note the achievement of Natanson?

It is the received view that Friedrich Hund (1896–1997) was the first historian of physics who noted achievements of Natanson in quantum statistics. However, it appears that in this respect, the priority belongs to Edmund Taylor Whittaker (1873–1956), an English mathematician, physicist, historian and philosopher of exact sciences, who at his *A History of the Theories of Aether and Electricity*. Vol. 2. *The Modern Theories 1900–1926* (1953) on pp. 88–89 states what follows:

Planck regarded the quantum property as belonging essentially to the interaction between radiation and matter: free radiation he supposed to consist of electromagnetic waves, in accordance with Maxwell's theory. Einstein in this paper put forward the hypothesis that parcels of radiant energy of frequency ν and amount $h\nu$ occur not only in emission and absorption, but that they have an independent existence in the aether. It was shown by P. Ehrenfest¹ of Leiden, by A. Joffe² of St Petersburg, by L. Natanson³ of Cracow and by G. Krutkow⁴ of Leiden that Einstein's hypothesis leads not to Planck's law of radiation but to Wien's, at any rate if we assume that each of the light-quanta or photons of frequency ν has energy $h\nu$ and that they are completely independent of each other. In order to obtain Planck's formula it is necessary to assume that the elementary photons of energy $h\nu$ form aggregates, or photo-molecules as we may call them, of energies $2h\nu, 3h\nu, \dots$, respectively, and that the total energy of radiation is distributed, on the average, in a regular manner between the photons and the different kinds of photo-molecules.

1. *Ann. d. Phys.* xxxvi (1911), p. 91 [Ehrenfest 1911].
2. *Ibid.*, p. 534 [Joffe 1911].
3. *Phys. ZS* xii (1911), p. 659 [Natanson 1911c].
4. *Phys. ZS* xv (1914), p. 133 [Krutkow 1914a].

Hence, in Whittaker's opinion, the essence of Natanson's achievement and of the other physicists: Ehrenfest, Joffe, and Krutkow was not discovery of Bose-Einstein statistics, but the fact they understood that

Einstein's hypothesis of independent photons led to Wien's law, and the "aggregates of photons" or "photo-molecules" led to Planck's law.

On the other hand, it is still valid thesis that only after Friedrich Hund monograph (1967) many researchers of history of quantum statistics discovered in this field the achievements of Natanson.

8. How many Natanson's works are known by the scholars writing about Natanson's contributions to the so-called Bose-Einstein statistics?

Nearly all scholars writing about Natanson's contributions to the so-called Bose-Einstein statistics know only the third paper on the list of Natanson's works given in Section 3.1 (Natanson 1911c), that is a German translation "Über die statistische Theorie der Strahlung" (published: 15 August 1911); they do not know of the first paper on this list (Natanson 1911a), that is his English paper "On statistical theory of radiation" (published: circa 10 April 1911).

The German translation was known and quoted, among others, by the following historians of physics: F. Hund (1967, p. 35 fn. 10; English transl. 1974; and Russian transl. 1980, p. 226 fn. 10), A. Hermann (1971, p. 28 fn. 34), J. Hendry (1980, p. 73 fn. 83), A. Kastler (1983, p. 623 fn. 9), O. Darrigol (1984, p. 659; 1988; 1993), A. Bach (1988; 1990), S.K. Das, S. Sengupta (1995), A. Kojevnikov ([2002](#), pp. 198 & 227), L. Navarro, E. Pérez (2004, p. 141), E. Garfield ([2004](#)), S. French, D. Krause (2006, pp. 91 & 404), S. Saunders (2006, [pdf version](#), p. 21; [2009](#), p. 304), M. Badino ([2009](#), pdf version, p. 26), A. Borrelli ([2009](#), p. 77), O. Passon, J. Grebe-Ellis ([2017](#), p. 7), etc.

This is also cited in the realistic interpretation of quantum mechanics proposed by A. Jabs (1996, p. 82, fn. 261).

Nevertheless, even this paper is omitted by several historians of physics writing about the genesis of the so-called Bose-Einstein statistics, such as H. Kangro (1970; English translation 1976), T.S. Kuhn (1978), and H. Kragh (2002).

However, the English and German versions of this paper are listed by M. Jammer (1966, p. 51 fn. 205; Russian translation: 1985, p. 60 fn. 205), M. Paty ([2001](#), p. 22), J. Stachel ([2000](#), pp. 245, 246, 251), S. Varró (2006a, pdf version, p. 33; 2006b, pdf version, p. 19; 2007, p. 169) and P. Enders ([2007](#), p. 87; [2009](#), p. 18).

These three papers (Natanson 1911a; 1911c; 1912b) are known also by the American historian of science (of the Polish origin) M.J. Konieczny (2008, [2010](#), [2011](#), [2012](#)), and the Japanese historian of physics N. Nagasawa (“Roh Minamida” 2009; Nagasawa [2018](#)).

The Polish scholars who write about Natanson’s contribution to the so-called Bose-Einstein statistics, such as B. Średniawa (1985, pp. 89–90; [1997](#), pp. 14–16; 2000, pp. 454–455; 2001, pp. 105–107; [2007](#), pp. 713–714, 721) and after him J. Spalek ([2005](#); [2006](#); 2009), know his other work, mentioned above in Section 4.1 and listed as the fifth position on this list – Natanson [1913](#): *Zasady Teorii Promieniowania* (The Principles of Theory of Radiation). *Prace Matematyczno-Fizyczne* 24, pp. 1–88. Warszawa: Wydawnictwo Redakcji Prac Matematyczno-Fizycznych. It is an extensive review article on the theory of radiation.¹²⁹ This rule does not apply to previous Polish researchers: J. Weysenhoff (1958) and A. Teske (1981) did not mention any of these three works; K. Szymborski (1980, p. 66) mentioned both English and German versions of Natanson’s article (1911a; 1911c); Bogdan Lange ([1992a](#); [1992b](#); 1997a; [1997b](#)), K. Czapla ([2005](#), p. 55, fn. 27) and A.K. Wróblewski ([2014](#), pp. 267, 273) only the German version (1911c).

In June 2011, the author of this article found out that another of Natanson’s paper dealing with our subject exists. It is titled “O promienianiu (On radiation)”. It was presented on the 19th of July in 1911 in Kraków during “XI Zjazd lekarzy i przyrodników polskich” (The 11th Congress for Polish Physicians and Natural Scientists), which was held between the 18th and the 22nd of July in 1911 (Natanson 1911b). In 1912 it was published as a reprint (Natanson 1912a). A revised version of this paper appeared in 1924 in Władysław Natanson’s book, *Oblicze*

¹²⁹ However, in the case of articles marked by me as “Natanson 1912b” (i.e. “On the Energy-content of material bodies”) and “Natanson 1913” (i.e. “Zasady Teorii Promieniowania” (“The Principles of Theory of Radiation”)), B. Średniawa provides incorrect information on several occasions. In the case of the first article, among others, the incorrect year 1911 of publication is consistently given. In the case of the second article, the incorrect year 1912 of publication and / or the number of pages are given, e.g.: “W. Natanson: Zasady teorii promieniowania. Wyd. Prac Matematyczno-Fizycznych. Warszawa 1912, 88 s.” (Średniawa [1997](#), p. 20) [88 pp. is the correct number]; “W. Natanson, *Prace Mat.-Fiz.*, t. 24 (1912), s. 1–352” (Średniawa 2001, p. 107) or “W. Natanson, *Mathematical and Physical Letters* 29, 1–232, Jagiellonian University 1912 (in Polish)” (Średniawa [2007](#), p. 721).

natury: odczyty, przemówienia i szkice (*The face of nature: lectures, speeches and essays*). Kraków: Krakowska Spółka Wydawnicza, [1924](#), pp. 125–153 (Natanson 1924b, i.e. Natanson [1924](#), pp. 125–153).

9. Conclusion: Why the name of Władysław Natanson was neglected for many years in the context of the so-called Bose-Einstein statistics?

Regarding the considerations made so far, I think that two matters worked here. Firstly, it was the lack of sufficient knowledge about primary sources, both letters and articles (it was the basis of the assumed view that Natanson's works on theory of radiation and related issues were entirely neglected and forgotten in 1910s and 1920s), and the second was the *Robert K. Merton effect*, that

[consisted] in the accruing of greater increments of recognition for particular scientific contributions to scientists of considerable repute and the withholding of such recognition from scientists who have not yet made their mark (R.K. Merton [1968](#), p. 3).¹³⁰

¹³⁰ However, as it is commonly known, R.K. Merton named it 'Matthew effect'. I think that this term was incorrectly chosen by this author, who while studying the *Parable of the Talents* of (Matthew 25:14–30), confused the divine perspective and the sphere of *sacrum* with the temporal perspective and the sphere of *profanum*, and formulated the principle according to which the "rich man" would only get richer (gain new talents), and the poor would be even more impoverished (lose talents). But, in Christian spiritual life, this rule simply does not apply, because (taking into account other passages from the Bible) this kind of "richness" seen by a human eye can appear to be the biggest poverty of a man. It suffices only to recall another biblical passage also from Matthew (19: 24): "It is easier for a camel to go through the eye of a needle, than for a rich man to enter into the kingdom of God" (cf. *Hebrew New Testament Studies* [2003](#)). Thus the *Parable of the Talents* is about something different: because God knows each heart, He passes a just judgment on our real intentions and activities and warns us against the consequences of bad deeds – cf. a Trapist monk Thomas Merton 1955 ([2005](#), p. 173); [1969](#), p. 16; Welchel [2013](#); *Wikipedia* [2019d](#).

In contrast, the readers of scientific works (papers, articles, books) – not possessing divine knowledge – do not know the real merits of various authors and co-authors and therefore judge them by appearances (unless the readers make a very detailed analysis of these achievements, *but in practice it is very difficult*). That's why they often pass sentences about these works and their authors that are far from the truth, e.g. exaggerated achievements of already famous scholars...

It needs to be highlighted that the effect joins three issues: a) the intellectual thefts and unreliability in quoting which unfortunately take place too often in science,¹³¹ b) the still dominant naïve interpretations of the history and philosophy of exact sciences based on the idea that a more famous scientist by definition and at once means more than important thinkers and their greatest merits and contributions in co-authored works,¹³² and c) the small or negligible final impact of authors caused by a lack of affiliation to a dominant scientific centre, a dominant science school, a dominant political faction or even a dominant nation.¹³³

At the same time, I do agree with the motto of the articles of Nagasawa (“Minamida” 2009; Nagasawa 2018) about the neglected achievement of Ladislas (Władysław) Natanson:

Citation is not only a working technique, but also an ethics, the acknowledgement of obligations and a respect for truth (Wolff 2003, p. 349).

Unlike many of his contemporary physicists, Natanson did not have any problems with this ethical and epistemological view.

¹³¹ This aspect is well known from bibliometric studies – cf. Kokowski 2015b, pp. 155–169.

¹³² Nobel Prize winners from various disciplines paid great attention to this aspect in interviews conducted by Zuckerman 1965; 1972; 1977. And this finding was used by R.K. Merton to formulate the term ‘Matthew effect’. It is worth remembering Merton’s own words on the dependence of his view on Zuckerman works: “This is occasion for repeating what I have noted in reprinting the original «Matthew Effect in Science» [R.K. Merton 1968 – M.K.]: «It is now [1973] belatedly evident to me that I drew upon the interview and other materials of the Zuckerman study to such an extent that, clearly, the paper should have appeared under joint authorship.»” (R.K. Merton 1988, p. 607 fn. 2).

¹³³ As regards point c), it is the reason that there is no common history of physics (or science), but there are many national histories of physics (or science) – cf. A.K. Wróblewski’s plenary lecture given during the 2nd International Conference of the European Society for the History of Science held in Cracow in 2006 entitled “Are we ready for common history of science?” (Wróblewski 2006).

Appendix 1. *Preliminary methodological considerations about the historical method*

Our comprehension of historical concepts, events and processes is possible thanks to the use of the historical method.¹³⁴

To explain this method I use the ideas of ‘understanding’ and ‘decoding’ of sources by using a deliberately chosen ‘hermeneutics’ i.e. ‘interpretative tools’, and the analogies of ‘a lens’, ‘a microscope’ or ‘a telescope’, and I say briefly:

Our understanding of historical sources consists in decoding their content using a specific hermeneutics (i.e. certain consciously chosen interpretative tools)¹³⁵. The aim of the historical method is to create certain narratives based on facts evidenced by historical sources and auxiliary studies. The reliability of these narratives depends on whether they ‘save historical phenomena’ (i.e. they are consistent with relevant selected historical events supported by the evidence in historical sources), and they are consistent also with the state of knowledge about the whole culture or some of its privileged parts.¹³⁶

The empirical basis of such narratives is found by analysing sources of historical information (handwritten documents, printed documents etc.). In order to read these documents with a proper understanding, we must be able to recognize handwritten characters (if we have to study

¹³⁴ For a description of this method see, for example, Droysen [1858](#); Freeman [1886](#); Bernheim 1889 (1st ed.; [1908](#) 5th ed.); Langlois, Seignobos 1898 (ed. of [1992](#)); Garraghan 1946; Gottschalk 1950; Butterfield [1955](#) (reissued 1969); (about German historicism) Iggers 1968 (rev. ed. 1983); Topolski 1968 (3rd corr. ed. 1984); 1978; 1983; 1998; Shafer 1974; McCullagh 1984; Firat [1987](#); Bentley (ed.) [1997](#); Bentley [1999](#); Phillips [2000](#); Howell, Prevenier [2001](#); Rüsen (ed.) 2006; Wood 2008; Edwards Education Blog [2013](#); Porra, Hirschheim; Parks [2014](#); Levitin 2015; Janssen 2017; Morgan [2017](#); Dagg [2019](#).

¹³⁵ For details see Kokowski [1999](#); [2001](#), pp. 5–9; [2007](#).

¹³⁶ I apply here an analogical reasoning – I mean the Hellenistic expression used in philosophy of nature ‘*Σώζειν τὰ Φαινόμενα*’ („to save phenomena”), and the ideas of ‘external confirmation’ and ‘inner perfection’ of a physical theory by Einstein; see Duhem 1908/1969 (reprinted 1985), and Einstein 1949, pp. 20–25. On the margin: this approach is versatile and more universal than the approach assumed in ‘general history’, ‘philosophical history’, ‘conjectural history’ (the latter developed by Montesquieu and Scottish scholars), and annals, memoirs, biography, and literary history – cf. Phillips [2000](#).

handwritten documents) and printed characters, as well as to recognize a linguistic dimension of the documents studied (the content of documents) we must possess proper linguistic abilities (in national languages, and languages of scientific disciplines).

In the historical approach, we can use a whole spectrum of different strategies (from antiquarianism¹³⁷ to presentism¹³⁸) and apply these strategies to analyse concepts (their content, genesis, and reception including a generation of new conceptual contents), events or historical processes from past to present times, and *vice versa* (I mean progressive studies or regressive ones) or at a particular time in the history.

However, the more we move away from the historical context of our times, learning about past events becomes more and more difficult. This is because they are mediated by the way these events were understood by direct witnesses, and subsequently how various categories of interpreters understood the witness statements on these events (including translators / linguists), and at the end, by ourselves, who, immersed in some thought collectives, are also interpreters of the past.¹³⁹

Moreover, our comprehension of historical events or processes, i.e. the result of applying the historical method, is always hypothetical, as it is based on many hypothetical premises. In researching and writing

¹³⁷ See Sweet [2008](#); Levitin 2015; Janssen 2017; the genesis of this approach is described in Momigliano [1990](#), chap. 3. “The rise of Antiquarian research”, pp. 54–79. “The essence of antiquarianism is a focus on the empirical evidence of the past, and is perhaps best encapsulated in the motto adopted by the 18th-century antiquary Sir Richard Colt Hoare, «We speak from facts, not theory.» Today the term is often used in a pejorative sense, to refer to an excessively narrow focus on factual historical trivia, to the exclusion of a sense of historical context or process” (*Wikipedia* [2019a](#)).

¹³⁸ Presentism is an anachronistic interpretation of the past from the point of view of the present-day ideas. See, for example, Fischer [1970](#), pp. 135–140; Syrjamäki [2011](#), pp. 20–49. However, in the strict sense of the word, one cannot completely avoid such a research perspective – see, for example, Fendler [2008](#); Spoerhase 2008.

¹³⁹ “The whole modern method of historical research is founded upon the distinction between original and derivative authorities. By original authorities we mean either statements by eye-witnesses, or documents, and other material remains, that are contemporary with the events which they attest. By derivative authorities we mean historians or chroniclers who relate and discuss events which they have not witnessed but which they have heard of or inferred directly or indirectly from original authorities. We praise original authorities – or sources – for being reliable, but we praise non-contemporary historians – or derivative authorities – for displaying sound judgment in the interpretation and evaluation of the original sources” (Momigliano [1950](#), p. 286).

about the historical past, apart from determining things, i.e. material documents of the epoch, dates and places of events or processes, it is also important to determine the names of people who played important roles (not necessarily major) in those events or processes. They are all needed to build a reliable story, written by a historian, which must persuade the audience to the picture described in such a story.¹⁴⁰

In the case of the historiography of sciences, we must also know documents (archival documents, published works, instruments, buildings, etc.), dates and places. As regards to scientists, we need to find not only names of discoverers, but also of their precursors and epigones. Regarding the story written by a historian of science, it must also be well-composed to persuade the audience to the picture described in such a story.¹⁴¹

This is all linked with several important issues that have been discussed for a long time by sociologists, historians of science, philosophers of science and intellectual historians. They include: the priority of discovery,¹⁴² multiple discoveries,¹⁴³ scientific precursors,¹⁴⁴ anachronism¹⁴⁵ including the Whig or Whiggish history of science and its overcoming by a detailed contextual research,¹⁴⁶ the still existing intriguing discrepancies in establishing by historians of science in different linguistic circles a coherent list of names of important scientific discoverers,¹⁴⁷ and finally the methodology of the historiography of sciences, which joins the methodology of historiography with the methodology of sciences.

While generalizing the problem of the historical narrative (both in historiography and historiography of sciences), it is worth distinguishing in every text (including a scientific text) three elements or strata: “*the form of the text* (the literary form of the text), *the hermeneutics of the text*

¹⁴⁰ Writing this story is linked with a more general problem of constructing the historical narrative by a historian – cf. Munz 1997; Topolski 1998.

¹⁴¹ About the history of the historiography of sciences cf. Agassi 1963; Kragh 1987; Markova 1987; Catana 2011. On Duhem’s return to sources, see Le Roux, Krasnodębski 2017, pp. 37–41.

¹⁴² R.K. Merton 1957.

¹⁴³ R.K. Merton 1961a; 1961b; 1963; Bikard 2013.

¹⁴⁴ Duhem 1908; 1913–1959; Langlois 1994, p. 1.

¹⁴⁵ Skinner 1969; Prudovsky 1997; Jardine 2000; Spoerhase 2008; Špelda 2012.

¹⁴⁶ Butterfield 1931; 1949 (2nd ed. with corrections 1957); Hyman 1988; Mayr 1990; Jardine 2003.

¹⁴⁷ Wróblewski 2007.

(that is all means applied explicitly or implicitly in the text to interpret the subject under study) and *the rhetoric of the text* (that is all means serving to convince the reader to the expounded theses”.¹⁴⁸

Regarding the methodology of the so-called exact sciences, we know that every theory belonging to these sciences is composed of a mathematical model and of quasi-entities defined in the context of this model. We also know that the hypothetico-deductive method of correspondence-oriented thinking (*Korespondenzdenken*) is a very useful tool for understanding the development of such sciences. The created new theories are often linked with earlier theories by correspondence principles and it is not accidental – it is a result of the application of the postulate of correspondence of subsequent theories by scientists. Every perfect theory must be logically and mathematically coherent and save phenomena. The latter is realized by using measuring instruments and correspondence rules linking observables and their representations postulated in the context of the theory.¹⁴⁹

Historians of science – in their investigations of the past science – have the freedom to choose any reasonable research hermeneutics. It can stem from the methodology of the so-called exact sciences, philosophy of science, sociology of scientific knowledge, etc. The main demand is epistemic coherence and fruitfulness of this approach, tested by decoding the previously encoded content of scientific publications.

Appendix 2. Comments regarding terminology: “Bose statistics”, “Bose-Einstein statistics”, “Einstein-Bose statistics” and “Planck-Bose statistics”

Shortly after the release of S.N. Bose’s articles: 1924a (reprinted [2009a](#); English transl. [2009b](#)); 1924b (reprinted [2009c](#); English transl. [2009d](#)), and then A. Einstein’s articles: 1924 (reprinted 2015a, [Doc. 283](#); Eng. transl. 2015b, [Doc. 283](#)); 1925a (reprinted 2015a, [Doc. 385](#); Eng. transl. 2015b, [Doc. 385](#)); 1925b (reprinted 2015a, [Doc. 427](#); Eng. transl. 2015b, [Doc. 427](#)), the terms “Bose statistics”, “Bose-Einstein statistics” or “Einstein-Bose statistics” (emphasizing the primary role of Einstein in

¹⁴⁸ About this understanding of the methodology of historiography of sciences cf. Kokowski [1999](#); [2001](#), pp. 5–9, 317; [2007](#).

¹⁴⁹ More about these aspects cf. Kokowski [1996](#); [2012](#); [2015c](#).

its genesis) and their equivalents in other languages than English were introduced in the recognition of the achievements of the authors of these works.

The term “Bose statistics” refers to the statistics that Bose introduced in the case of blackbody radiation (the quanta of light, called photons after 1926),¹⁵⁰ and the terms “Bose-Einstein statistics” or “Einstein-Bose statistics” to the new quantum statistics, which was generalized by Einstein also for the case of ordinary matter particles (perfect gas).

- 1) “Bosesche Statistik”: Adolf Smekal to Albert Einstein (Vienna, 5 February 1925; cf. Einstein 2015a, Doc. 434, p. 644); Max Born to Albert Einstein (Göttingen, 15 July 1925; cf. Einstein 2018, Document 23, pp. 70–72, here p. 70); Pascual Jordan to Albert Einstein (Göttingen, 27 October 1925; cf. Einstein 2018, Document 98, p. 177); Smekal 1926, p. 319; Jordan 1927, p. 637; Jordan, Wigner 1928, p. 635/Wigner 2013, p. 113; Haas 1928, pp. 114, 119, 120, 121, 122, 124, 126, 129; Heitler, Herzberg 1929; Fierz 1939, pp. 3, 28.
- 2) “Bose statistics”: Weisskopf 1939, pp. 72, 75, 82 / 1958, pp. 68, 71, 78; Pauli 1940, p. 13; Feynman 1949, pp. 773, 782; Feynman 1950, p. 452 / 1958, p. 269; Kubo et al. 1965, p. 29. fn. †; Sudarshan 1968, p. 379; Sudarshan 1974/1975, p. 70.
- 3) “Статистика Бозе”: Leontovich 1944, pp. 174, 175, 176; Landau, Lifshitz 1937–1939 (ed. 1975, p. 180) / (English transl.) 1958, p. 153).
- 4) “Bose-Einsteinischen Statistik”: Smekal 1925, p. 613; Born, Heisenberg, Jordan 1926 (received 16 November 1925), p. 609 [reprinted in: Heisenberg 1985, p. 449]; Albert Einstein to Paul Ehrenfest (Berlin, 24 November 1926; cf. Einstein 2018, Document 420, pp. 644, 645); Jordan 1927, p. 637; Wigner, Witmer 1928, p. 868 / Wigner 2013, p. 176; Schaefer 1937, pp. 30, 405 fn. 411; Schäfer 1950, pp. 208, 209.
- 5) “Statystyka Bosego-Einsteina”: Skłodowska-Curie 1939, p. 98.
- 6) “Статистика Бозе – Зйнштейна”: Leontovich 1944, pp. 174, 176; Frenkel’ 1948, p. 637, 643, 644.

¹⁵⁰ According to the current views, the term “photon” originated in 1916, but gained acceptance after the article by Gilbert N. Lewis (1926). On the genesis of this term and its meaning see Kragh 2014 and Hentschel 2018.

- 7) “Bose-Einstein statistics”: Feynman [1950](#), p. 454 / [1958](#), p. 271.
- 8) “Einstein-Bose statistics”: Wigner 1926, p. 492 / 2013, p. [34](#); Dirac [1927](#), pp. 245, 247, 250, 251, 253, 255, 260 / [1958](#), p. 3, 5, 8, 9, 11, 13, 18; Fowler [1929](#), pp. 537, 543, 553, 556, 557, 559; Rasetti [1929](#), p. 516; Kemmer 1938, p. 127; Pauli [1940](#), p. 13; Lindsay 1941 ([6th printing 1962](#), p. 193).
- 9) “La statistique de Einstein-Bose”: Born, Heisenberg, Langevin, Kramers, Dirac (cf. Institut international de physique Solvay [1928](#), pp. 175, 176, 269, 270, 271, 272; Pauli [1936](#)).

Moreover, some researchers assumed that the terms “Bose statistics” and “Bose-Einstein statistics” are only different names for the same statistics called “Bose statistics” or “Bose-Einstein statistics” – Landau, Lifshitz 1937–1939 (ed. [1975](#), p. 180) / English transl. [1958](#), p. 153; Leontovich [1944](#), pp. 174, 175, 176); Kubo et al. [1965](#), p. 29 fn. † / Russian transl. [1967](#), p. 43 fn. 2.

After the article by Wolfgang Pauli ([1940](#)) it was known that the statistics of BE describes particles with spin 0, 1, 2, ... Taking this into account and acknowledging S.N. Bose’s merits, P. Dirac (1945 [cited after Farmelo 2009, p. 331, n. 64], [1947](#)) named particles with spin 0, 1, 2, ... bosons.

The new statistics was first studied by Bose, so we shall call particles for which only symmetrical states occur in nature bosons. [...] We can see the difference of Bose statistics from the usual statistics by considering a special case – that of only two particles and only two independent states a and b for a particle. [...] Thus with Bose statistics the probability of two particles being in the same state is greater than with classical statistics. Bose statistics differ from classical statistics in the opposite direction to Fermi statistics, for which the probability of two particles being in the same state is zero. [...] Planck’s law of radiation shows us that photons are bosons, as only the Bose statistics for photons will lead to Planck’s law (Dirac [1947](#), [3rd rev. ed.](#), pp. 210–211).

However, in the first edition of Dirac’s monograph of [1930](#) he applied a more general term: “Einstein-Bose statistics”:

This statistical mechanics is known as the Einstein-Bose statistics, as it was first introduced by Bose and Einstein before the arrival of the modern quantum mechanics (Dirac [1930](#), p. 201; cf. also p. 219).

Currently, the dominant term is the term “Bose-Einstein statistics” (Google: 153,000) and its equivalents in other languages: “Статистика Бозе-Эйнштейна” (58,600), “Statistique de Bose-Einstein” (21,000), “Bose-Einsteinschen Statistik” (201), “Statystyka Bosego-Einsteina” (470). The term replaced the term “Einstein-Bose statistics” (Google: 3,460) and its language equivalents: “statistique de Einstein-Bose” (81), “Einstein-Bosesche Statistik” (7), “statystyka Einsteina-Bosego” (1).

The term “Bose statistics” is still in use (Google: 26,700) and its language equivalents: “Статистика Бозе” (2,430) “statistique de Bose” (1,050), “statystyka Bosego” (86), “Bosesche Statistik” (15).

Moreover, John Hendry ([1984](#), p. 70) uses the term “Planck-Bose statistics” instead of the term “Bose statistics” (only two cases in Google linked with Hendry’s book).

Appendix 3. Letter from Max Planck to Władysław Natanson (25 January 1913)

There is very interesting evidence from the beginning of 1913 that Planck appreciated Natanson as a physicist. This is Sommerfeld’s letter from 25 January 1913 sent from Berlin-Grunewald to Natanson in Kraków:

I attach below a transcription of this letter with its translation into English.

Berlin-Grunewald, 25.I.13

Hoch verehrter Herr College!

Nehmen Sie meinen besten Dank für Ihnen freundlichen Brief von 21. [...] Selbstverständlich werde ich mir eine besondere Freude daran machen, Sie als Mitglied der Deutscher Physikalischen Gesellschaft vorzuschlagen. Die Aufnahme wird voraussichtlich in 4 Wochen erfolgen,

und Ihnen alsbald von Bureau der Gesellschaft angezeigt werden.

Mit verbindlichen Grüßen
Ihr ergebenster
M. Planck

60

Berlin-Grunewald, 25. I. 13.

Hochverehrter Herr College!

Nehmen Sie meinen besten Dank für Ihren freundlichen Brief vom 21. d. M. Selbstverständlich werde ich mir eine besondere Freude daraus machen, Sie als Mitglied der Deutsch Physikalischen Gesellschaft vorzuschlagen. Die Aufnahme wird voraussichtlich in 4 Wochen erfolgen, und Ihnen alsbald vom Bureau der Gesellschaft angezeigt werden.

Mit verbindlichen Grüßen
Ihr ergebenster
M. Planck.

Fig. 16. Sommerfeld's letter to Natanson (25 January 1913). Source: © Biblioteka Jagiellońska, photo: © Michał Kokowski.

Berlin-Grunewald, 25 January, 1913

Esteemed Colleague!

Please receive my great thanks for your kind letter of 21. [illegible] It goes without saying that it will be a great joy

to recommend you as a member of the German Physical Society. The admission is expected to take 4 weeks, and will be announced to you by the Society's Bureau.

With binding regards
Your most devoted
M. Planck

Appendix 4. Excerpts of two letters from Sommerfeld to Rubinowicz (Stockholm, 1 October 1919; München, on 1 November 1919). Transcription and translation

In the letter from Stockholm, on 1 October 1919 – see Sommerfeld [1919a](#) (archival document) – we read on p. 1/2 as follows:

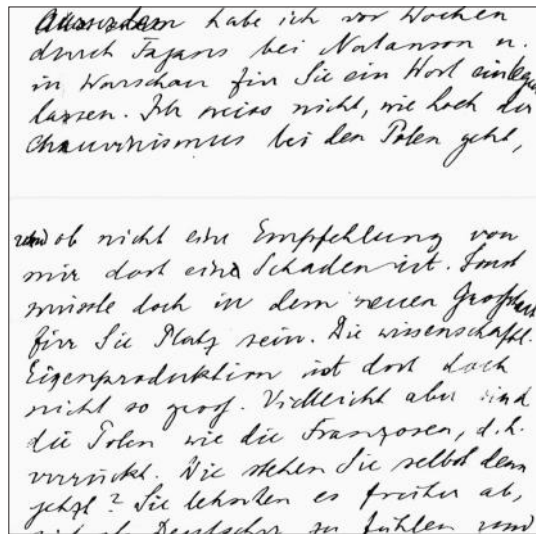


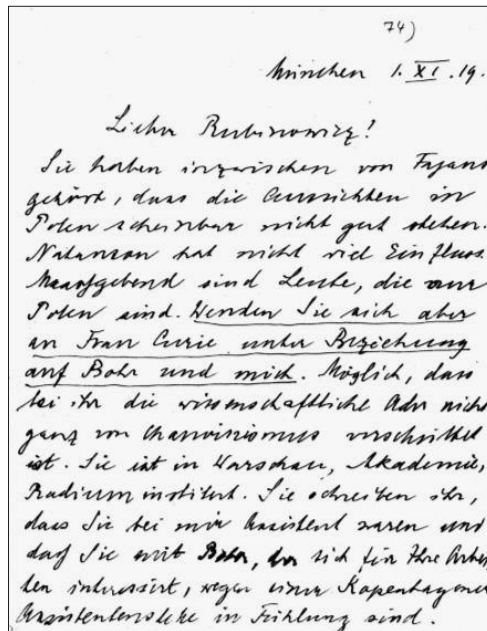
Fig. 17. The excerpt of the letter from Sommerfeld to Rubinowicz (Stockholm, 1 October 1919). Source: Sommerfeld [1919a](#).

Ausserdem habe ich vor Wochen durch Fajans bei Natanson u. in Warschau für Sie ein Wort einlegen lassen. Ich weiss nicht, wie hoch der Chauvinismus bei den Polen geht, und ob nicht eine Empfehlung von mir dort ein

Schaden ist. Sonst würde doch in dem neuen Grossstadt für Sie Platz sein. Die wissenschaftl.[ische] Eigenproduktion ist dort doch nicht so gross. Vielleicht aber sind die Polen wie die Franzosen, d.h. [das heißt] verrückt. Wie stehen Sie selbst denn jetzt? [Sommerfeld 1919c ([archival document](#)), p. 1; transcription – M.K.; F.K.].

Additionally, a few weeks ago I left a word for you via Fajans at Natanson's as well as in Warsaw. I do not know how much chauvinism the Poles hold, and whether a recommendation from me would not cause a problem. Otherwise, there would be a place for you in a new big city. Their own scientific production is not so large. Yet, perhaps the Poles are like the French, that is crazy. What would be your stance now? [translation – M.K.].

In the letter of München, on 1 November 1919 – see Sommerfeld 1919c ([archival document](#)) – we read on p. 1 as follows:



74)

München 1. XI. 19.

Lieber Rubiniowicz!

Sie haben inzwischen von Fajans gehört, dass die Aussichten in Polen schon eher nicht gut stehen. Natanson hat nicht viel Einfluss ausgeübt und Leute, die nach Polen sind. Werden Sie sich aber an Frau Curie unter Bezeichnung auf Bohr und mich. Möglich, dass bei ihr die wissenschaftliche Arbeit nicht ganz von dem wissenschaftlichen Vorwissen ist. Sie ist in Warschau, Akademie, Radziwiłł-Institut. Sie schreiben mir, dass Sie bei mir herzlich waren und dass Sie mit Bohr, der sich für Ihre Arbeiten interessiert, wegen einer Kapazitätsgrenze Assistentenstelle in Feilung sind.

Fig. 18. The excerpt of Sommerfeld's letter to Rubiniowicz (München, on 1 November 1919). Source: Sommerfeld 1919c ([archival document](#)).

Liber Rubinowicz,

Sie haben inzwischen von Fajans gehört, dass die Aussichten in Polen scheinbar nicht gut stehen. Natanson hat nicht viel Einfluss. Massgebend sind Leute, die nur Polen sind. Wenden Sie sich aber an Frau Curie unter Beziehung auf Bohr und mich. Möglich, dass bei ihr die wissenschaftliche Adr [Adresse] nicht ganz von Chauvinismus verschriftet ist. Sie ist in Warschau, Akademie, Radiuminstitut [sic! – in fact, she was in Paris]. Sie schreiben ihr, dass Sie bei mir Assistent waren und dass Sie mit Bohr, der sich für Ihre Arbeiten interessiert, wegen einer Kopenhagener Assistentenstelle in Fühlung sind [Sommerfeld [1919c](#) (*archival document*), p. 1; transcription – M.K.; F.K.].

Dear Rubinowicz,

In the meantime you have already heard from Fajans that the prospects for a job in Poland are not good. Natanson does not have much influence. Only people who are Polish have some authority. However, contact Ms. Curie and refer her to Bohr and me. It is possible that her scientific stance is not entirely chauvinistic. She is in Warsaw, Academy, Radium Institute [sic! – in fact, she was in Paris]. Write to her that you worked as an assistant with me and that you are in touch with Bohr, who is interested in your work regarding an assistant position in Copenhagen [translation – M.K.].

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