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A Bridge in the History of Science: The Ṭūsī Couple and Copernicus's Geometric Legacy

Abstract

Since the late 20th century, a number of discoveries in the history of Islamic science have prompted a re-evaluation of the origins of the Scientific Revolution. The prevailing criticism of the scientific revolution concept, as it pertains to Copernicus, Galileo and Newton, is that it disregards the historical context. Given the similarity between the sources of Islamic and European scientific traditions, it is implausible that the new ideas were independent of each other. This suggests that scientific change may have preceded the advent of modern science.

The present study aims to shed light on this by examining the influence of Nasīr al-Dīn al- Ṭūsī's (1201–1274) geometrical model, known as the "Ṭūsī couple", on the work of Copernicus. The study demonstrates remarkable parallels between al- Ṭūsī and Copernicus, thereby indicating that this is not merely a matter of chance. The Ṭūsī couple's utilisation of both observation and mathematics proved to be a pivotal factor in the development of Copernicus's heliocentric model of the universe. The direct influence of the Ṭūsī couple on Copernicus's *De Revolutionibus Orbium Coelestium* has led to a reconsideration of narratives of the origins of the Scientific Revolution that emphasise the importance of scientific interaction. This suggests that there was a scientific interaction between the Islamic world and European thought, and that this interaction needs to be addressed in a broader framework.

Keywords: *Scientific Revolution, Nasīr al-Dīn al- Ṭūsī, Nicolaus Copernicus, Ṭūsī-couple*

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Pomost w historii nauki: Para al-Ṭūsīego a geometryczne dziedzictwo Kopernika

Abstrakt

Od końca XX wieku szereg odkryć w dziedzinie historii nauki islamskiej skłonił do ponownej oceny źródeł rewolucji naukowej. Główny zarzut wobec koncepcji rewolucji naukowej – w odniesieniu do dokonań Kopernika, Galileusza i Newtona – dotyczy pomijania kontekstu historycznego. Biorąc pod uwagę podobieństwa między źródłami tradycji naukowych świata islamu i Europy, mało prawdopodobne jest, by nowe idee powstawały niezależnie od siebie. Sugeruje to, że przemiany naukowe mogły poprzedzać narodziny nowożytnej nauki.

Niniejsze opracowanie ma na celu naświetlenie tego zagadnienia poprzez analizę wpływu modelu geometrycznego Nasīra al-Dīna al-Ṭūsīego (1201–1274), znanego jako „para al-Ṭūsīego", na twórczość Kopernika. Badanie ukazuje uderzające podobieństwa między rozwiązaniami al-Ṭūsīego a koncepcjami Kopernika, wskazując, że nie jest to dziełem przypadku.

Wykorzystanie przez al-Ṭūsīego zarówno obserwacji, jak i matematyki, okazało się kluczowym czynnikiem w procesie tworzenia przez Kopernika heliocentrycznego modelu wszechświata. Bezpośredni wpływ „pary al-Ṭūsīego" na dzieło Kopernika *De revolutionibus orbium coelestium* skłonił do rewizji narracji dotyczących źródeł rewolucji naukowej i podkreślenia znaczenia wzajemnych oddziaływań w nauce. Wskazuje to na istnienie wymiany naukowej między światem islamu a myślą europejską oraz na konieczność rozpatrywania tych relacji w szerszym kontekście.

Słowa kluczowe: *rewolucja naukowa, Nasīr al-Dīn al-Ṭūsī, Mikołaj Kopernik, para al-Ṭūsīego*

1. Introduction: Ṭūsī-Couple: A New Perspective in Celestial Mechanics

The Islamic astronomical tradition sustained a five-hundred-year philosophical-physical critique of Ptolemy's geocentric model, the central target of which was the equant. The equant[1] was a geometrical device that required planetary motion to appear uniform not around the actual centre of the orbit, but around a displaced point. This arrangement violated the Aristotelian principle of uniform circular motion, which was accepted by virtually all astronomers in the ancient and medieval period.[2] It was within this critical tradition – not as a departure from philosophy but as a rigorous engagement with it – that al-Bīrūnī advanced methodological standards for observational astronomy and Alī Qūshjī clarified the epistemological autonomy of mathematical model-building from Aristotelian natural philosophy. This standpoint signifies a particular methodological disposition within an ongoing discourse, as opposed to a novel disciplinary innovation. It is precisely this intellectual lineage that gave rise to, and facilitated the comprehension of, the mathematical pioneering work of Naṣīr al-Dīn al- Ṭūsī.[3]

One of the most significant models is the mathematical model of intertwined circles discovered by Nasīr al-Dīn al- Ṭūsī, known as the 'Ṭūsī-couple'. The Ṭūsī-couple was conceptualised as a rigorous geometric response to the equant problem in Ptolemaic astronomy. Specifically, the physically problematic requirement that the center of the epicycle move at a uniform angular velocity around a point distinct from the geometric center of the deferent. By combining two nested circles whose radii stand in a ratio of 1/2 (the radius of the inner circle being exactly half that of the outer circle) and which rotate at defined, inversely related angular velocities, the Ṭūsī-couple successfully generates a rectilinear oscillation exclusively from uniform circular motions. This mechanism effectively eliminates the need for the equant device while fully retaining the physical and philosophical legitimacy of uniform circular motion within celestial mechanics. The Ṭūsī-couple is not merely a single tool or model; rather, it is

[1] The equant is identified as "having the reference point for uniform motion of an orb being different from the actual centre of the orb" (Ragep 2022, p. 4).

[2] *Ibidem*.

[3] Ragep 2022, pp. 32, 34.

a comprehensive geometrical approach employed to address various astronomical problems. The initial version of the Ṭūsī-couple comprises two concentric circles of differing sizes and rotational velocities, capable of generating linear motion in a plane between two points.[4] The smaller circle, with a diameter equivalent to half that of the larger circle, rotates in such a manner that it makes contact with the larger circle at a point from the interior (see Fig. 1).

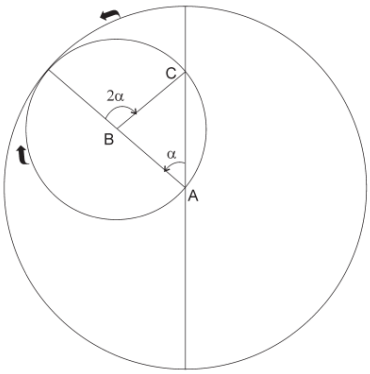


Fig. 1. "Mathematical rectilinear version of the Ṭūsī-couple"[5]

Consequently, the Ṭūsī-couple provides a physically coherent account of planetary motion by deriving all observable motions exclusively from uniform circular components. This is in contrast to Ptolemy's equant-based models, which achieved equivalent observational accuracy at the expense of physical principles.[6]

Ṭūsī conceptualised the circles employed to elucidate the movements of celestial bodies as both circles and solid spheres, or celestial orbits. In this approach, the motion of the smaller circle is indispensable for the manifestation of linear motion, and it is evident that the larger circle carries the smaller one.[7] This transformation into solid spheres represents the second version of the Ṭūsī-couple model, termed the 'physicalized rectilinear version' (see Fig. 2). This approach enables a more physical explanation of the motions of celestial bodies.

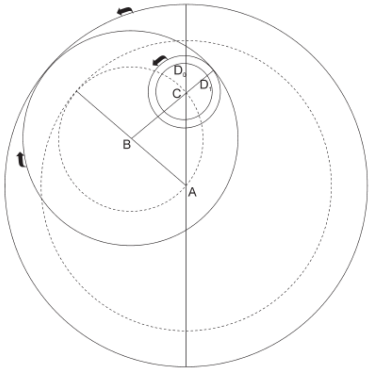


Fig. 2. "Physicalized rectilinear version of the Ṭūsī-couple"[8]

Ṭūsī utilised this version as an alternative model for Ptolemy's lunar model. Subsequently, in the *Tadhkira*, Ṭūsī made some modifications to the lunar model first presented in *Hall*.[9] This

[4] Ragep 2022, p. 66.
[5] Ragep 2022, p. 67.
[6] Blåsjö 2018, p. 484.
[7] Ragep 2022, p. 66.
[8] Ragep 2022, p. 68.
[9] *Ibidem*.

demonstrates that the rectilinear version of the Ṭūsī-couple was developed over time and in certain stages. Indeed, the refinement of such complex scientific models generally necessitates a protracted process, as evidenced by the emergence of a scientific model or theory as a perfect entity at once being incompatible with the nature of scientific discoveries.

Recognising that celestial motions are carried by solid spheres in three-dimensional space — the configuration captured by the spherical version of his device — why did Ṭūsī also develop the simpler, plain (planar) version of his pair?[10] The response to this enquiry is intimately associated with the scientific comprehension of his era and the model development process. In this context, Ṭūsī''s primary motivation, as documented in his own astronomical writings, was to restore uniform circular motion to the heavens — a principle that Ptolemy's equant had violated — by constructing geometrically rigorous alternatives that were physically coherent within the Aristotelian framework.[11] The rectilinear version of the Ṭūsī couple is, in this sense, an important example of how scientific discoveries generally reflect the gradual and step-by-step process of progress.[12]

About fifteen years after publishing the first version of his couple, Ṭūsī introduced a new model consisting of three celestial spheres surrounding the epicycles and intended to produce a curvilinear oscillation (see Fig. 3).

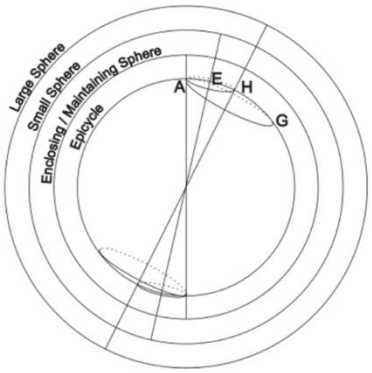


Fig. 3. Complete curvilinear version of the Ṭūsī-couple[13]

Crucially, while the mathematical construction of the curvilinear version is precise, Ṭūsī's verbal exposition suffers from a technical inaccuracy, as demonstrated by Ragep.[14] This astronomical model developed by Ṭūsī is a modification of the Eudoxus Couple that Ibn al-Haytham had worked on earlier (see Fig. 4).

[10] Ragep 2022, p. 72.
[11] Ragep 2022, p. 4.
[12] Such developments we can describe using the idea of scientfic (r)evolutions, general correspondence principles and mathematical linits – see Kokowski 2004; 2012.
[13] Ragep 2022, p. 74.
[14] Ragep 2022, p. 129.

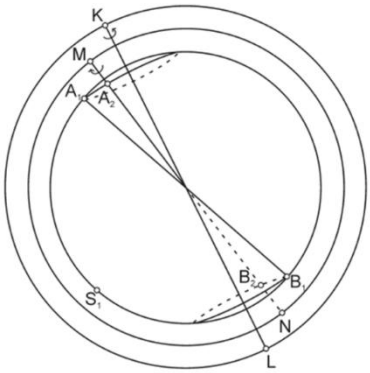


Fig. 4. Ibn al-Haytham's Eudoxan-couple, showing two spheres[15]

This curvilinear version of Ṭūsī's couple offers a simpler and more comprehensible alternative to the Eudoxan-Couple, also known as the complex 'system of concentric spheres' developed by Ptolemy.[16] Ṭūsī's curvilinear version is a versatile model that can be used to explain many different celestial phenomena, such as the irregularities of the Moon, the motions of the planets, the Earth's precession, and the tilt of its axis.

The phenomenon of the systematic under-acknowledgement of non-European contributions in Western scientific historiography is not unique to the Ṭūsī couple model. Within the history of optics, Ibn al-Haytham's *Kitāb al-Manāẓir*, which reformulated the intromission theory of vision on rigorous experimental and geometrical foundations of medieval and early modern optics, was disseminated to the Latin West through its late twelfth- or early thirteenth-century translation, *De Aspectibus*. Roger Bacon, Witelo, John Pecham, and ultimately Johannes Kepler built upon this perspectivist tradition; the Arabic foundations of that tradition, however, were long overlooked in standard European accounts, which privileged the Latin intermediaries over the original source.[17] This dynamic exemplifies a broader structural characteristic of intercultural transmission that constitutes the focus of the present paper: as scientific knowledge crosses cultural boundaries and becomes progressively integrated into the receiving tradition, the prominence of its historical origins tends to diminish in tandem.

2. Before Copernicus: A Link between East and West in Astronomy

The evolution of the various iterations of the Ṭūsī-couple extends over a period of approximately twenty-five years, yet no substantiation has been identified for the translation of this text into non-Islamic languages during the pre-Copernican era (1473–1543). Therefore, networks of cultural interaction are often invoked to argue that the appearance of the couple in other contexts is due to cross-cultural transmission.[18] The existence of such transmission in the history of science is not necessarily predicated on written evidence. The structural identity between the mechanisms employed by Ṭūsī and those found in Copernicus's De Revolutionibus, combined with the documented circulation of Maragha-school models through late Byzantine intermediaries, provides strong circumstantial evidence for direct or indirect

[15] Ragep 2022, p. 76.

[16] Ragep 2022, pp. 73–74.

[17] Lindberg 1976, p. 104. In this seminal work, Lindberg demonstrates that the Latin perspectivist lineage – Bacon, Witelo, Pecham – constituted the primary conduit through which Ibn al-Haytham's intromission theory ultimately informed Kepler's optics, while the Arabic foundations of this synthesis long remained marginalised in standard European historiography.

[18] Ragep 2022, pp. 75–77.

influence. The hypothesis of independent discovery, while not logically impossible, has yet to be substantiated by positive evidence.[19] Willy Hartner (1905–1981) advanced the hypothesis that Copernicus had borrowed the concept from Ṭūsī, but this claim remains unproven, as no specific transmission route has been identified.[20]

The Scientific Revolution, which played a major role in the emergence of the modern world, is widely regarded as one of the most significant turning points in the history of science. Nevertheless, this revolution has been regarded as a European-centred process, with emphasis placed on its foundation in ancient Greek philosophy and European Renaissance thought. This perspective, however, disregards the fact that the Scientific Revolution was a global phenomenon, and that scientific contributions from diverse cultures, including the Islamic world, were integral to this process.[21] Despite assertions that Copernicus and his contemporaries may have been influenced by Islamic astronomers, it is notable that the historical context of these models within European astronomy is weak.[22]

In contrast, the historian of mathematics Viktor Blåsjö has argued that the model presented by Copernicus was an independent discovery. Blåsjö has put forward a more cautious interpretation of Copernicus's widely quoted phrase "as some have called it" (*ut aliqui appellaverunt*) in *De Revolutionibus* III.4. According to Blåsjö, the referent of the demonstrative in the surrounding passage is not the Ṭūsī-couple as a mechanical device, but rather the rectilinear libration—the simple harmonic oscillation—produced by this device along the diameter of the outer circle. This distinction is significant: if Copernicus was alluding to harmonic motion as a phenomenon already familiar in pre-Copernican European natural philosophy, the passage cannot, when considered in isolation, function as direct textual evidence of his knowledge of the Marāgha tradition.[23] This interpretative shift does not, however, entail a denial of the technical parallels between the two traditions. Blåsjö neither contests the mathematical equivalence between Copernicus's two-circle construction and that of Ṭūsī, nor minimises the role of the generating circles in producing the resultant oscillation. Indeed, he explicitly acknowledges that the two constructions share the same geometrical starting point and the same kinematic objective. His critique is interpretative and philological rather than mathematical: it concerns the identification of the unnamed *aliqui* rather than the structural identity of the mechanisms themselves.

The methodological force of Blåsjö's caution against converting ambiguous textual references into transmission proofs is, in our view, well taken. However, textual indeterminacy alone cannot support an inference to independent convergent discovery. Three strands of historical evidence are mutually reinforcing: the Byzantine channel mediated by Gregory Chioniades, the Cracovian milieu of Copernicus's early astronomical formation, and his Italian sojourn at Bologna, Padua, and Ferrara—each of which is examined in detail in the following sections. Consequently, cultural transmission renders the more parsimonious account. When the structural identity of the mechanisms is considered in relation to the cumulative weight of these channels, the hypothesis of independent convergent discovery would require precisely the kind of positive evidence – biographical, codicological, or doctrinal – that has not, to date, been adduced. The assertion that the case for transmission has been rendered moot is refuted by Blåsjö's interpretative intervention, which serves to refine the argument by shifting the evidential burden from the contested *aliqui* passage to the broader codicological and prosopographical record.

[19] Hartner 1973: pp. 420–421.
[20] Nosonovsky 2018, p. 1.
[21] Bala 2016, p. 63.
[22] Bala 2016, p. 76.
[23] Blåsjö 2018, p. 484.

A tradition of systematic criticism of Ptolemaic cosmology in Europe would be expected, but this tradition has an unbroken precedent of five hundred years in the Islamic world. The necessity for Copernicus to formulate a novel cosmological model, a consequence of the inconsistencies within the Ptolemaic system and its incompatibility with observational data, serves as a primary motivation for the aforementioned change. This prompts the question of whether there is a link between Copernicus's work and astronomical studies in the Islamic world. In his seminal work *De Revolutionibus*, Copernicus employed a version of Ṭūsī's well-renowned invention, designated as Ṭūsī's couple by Kennedy (1966),[24] to elucidate the disparity in planetary precession and the variations in the ecliptic inclination. The model depicted in the figure is derived from a manuscript by Copernicus held in the library of the Jagiellonian University in Cracow (see Fig. 5).

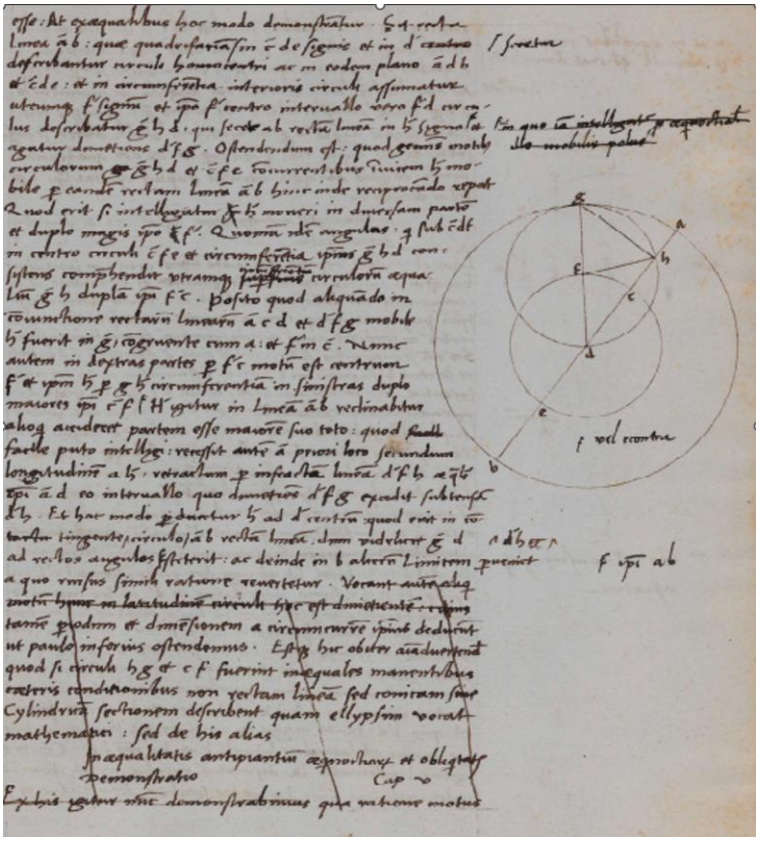


Fig. 5. The model used by Copernicus (Ṭūsī-couple)[25]

Copernicus employed the Ṭūsī-couple in both his *Commentariolus* and *De Revolutionibus* to elucidate the oscillation of orbital planes in the theory of the planets. Specifically, the change in the radius of the orbit of the planet Mercury was modelled by Copernicus using the parallel-axis form of the Ṭūsī-couple.[26] This demonstrates that Copernicus utilised the Ṭūsī-couple with great flexibility to address various astronomical problems. Copernicus's preference for the Ṭūsī-couple, which is a mathematical model rather than physical models to explain the motions of celestial bodies, shows that this tool can provide a flexible solution to astronomical problems. It is crucial to emphasise that mathematical discoveries are inherently dependent on historical contact or transmission; Copernicus's familiarity with the Ṭūsī-couple serves as a significant testament to this fact.

[24] Kennedy 1966, p. 369.
[25] Copernicus, *De Revolutionibus Orbium Coelestium*, p. 163.
[26] Swerdlow, Neugebauer 1984, pp. 46–47.

The mathematical models in Copernicus's works demonstrate striking parallels with the geometric models and trigonometric calculation methods developed by the Meraga School, thereby reinforcing the likelihood of Copernicus having direct or indirect access to Islamic astronomy. In particular, concepts such as the Ṭūsī-couple and the system of concentric spheres are among the most prominent examples of this interaction. Researchers such as Swerdlow and Neugebauer agree with this view and ask more detailed questions about when and how this information reached Copernicus, beyond the question of whether Copernicus learned the Meraga theory.[27] The transmission of this knowledge, direct personal contacts, translations, cultural exchange through the travels of scientists and trade may have played an important role in this transmission. The following discussion aims to shed light on the potential channels through which such transmission may have occurred.

According to the historian of science Fuat Sezgin (1924–2018), the transmission of the planetary motions to Copernicus did not occur via Latin translations, but rather via Byzantium, from Tabriz and Meraga via Trabzon and Istanbul. In particular, Polish scholars such as Sandivogius of Czechel (1430) and Adalbertus of Brudzevo (1482), who wrote commentaries on Gerhardus' "*Theorica Planetarum",* and Peurbach's *"Theoricæ Novæ Planetarum",* were well acquainted with the planetary theories of the Arab-Islamic cultural environment.[28] This suggests that the Ṭūsī-couple, an important mathematical concept used in Islamic astronomy, was known in Cracow, an important scientific centre in Poland during Copernicus's lifetime.

The historian of science Jamil Ragep contends that linguistic and cultural factors played a pivotal role in the dissemination of the Ṭūsī-couple to the West. Ragep's argument is that the concept was transmitted to Byzantium through the intermediary of Gregory Chioniades, who worked with Shams al-Din al-Wābkanawī. It is asserted that al-Wābkanawī travelled to Tabriz around 1295, translating from Persian into Greek. Chioniades' contributions were met with acclaim, leading to his integration into the court's intellectual circles. He subsequently returned to Trabzon with a significant collection of astronomical texts that he had translated. Research has demonstrated that the Ṭūsī-couple and other astronomical concepts based on this model in Chioniades' *Schemata* were transferred to the West. Presently, three manuscripts of the '*Schemata*' are extant: Two of these are housed in the Vatican Library, while a third is located in the Biblioteca Medicea Laurenziana in Florence. Swerdlow and Neugebauer favour this route of transmission in explaining the transmission of the Ṭūsī-couple to Copernicus, who studied and travelled in Bologna, Padua and Rome between 1496 and 1503.[29]

There is compelling circumstantial evidence to suggest that Copernicus was influenced by the work of Ṭūsī. Willy Hartner's comparisons provide substantial evidence for striking similarities between the geometrical diagrams in Copernicus's works and the drawings in Ṭūsī's "*Tadhkira fi 'ilm al-hay'a*". Hartner's observations indicate that both scientists employed a comparable approach in their utilisation of symbols and the depiction of movements in geometrical diagrams. This lends further credence to the hypothesis that Copernicus was influenced by Ṭūsī's study.

However, Blåsjö has advanced an opposing argument, proposing that the geometric notation employed in Copernicus's diagrams adheres to the sequence of Euclid's propositions.[30] Blåsjö's own formulation explicitly acknowledges that

> Copernicus and his Islamic predecessors had the same starting point, the same goal, and the same techniques at their disposal; the dispute concerns the mechanism of transmission, not the mathematical identity of the models (Blåsjö 2018, p. 480).

[27] Ragep 2022, p. 86.
[28] Sezgin 2007, p. 54.
[29] Ragep 2022, pp. 89–91.
[30] Blåsjö 2018, p. 486.

In this context, when the pre-Copernican references were analysed, the manuscript of the French scientist Nicole Oresme titled *"De Spera"* attracted much attention. In this work, Oresme combined three uniform circular motions and explained how reclinear motion could be produced.[31] The mechanism in question bears a striking resemblance to the Ṭūsī-couple (see Fig. 6). This finding suggests that Oresme's work provides significant insights into the dissemination of the Ṭūsī-couple in Europe and its influence on prominent astronomers, including Copernicus.

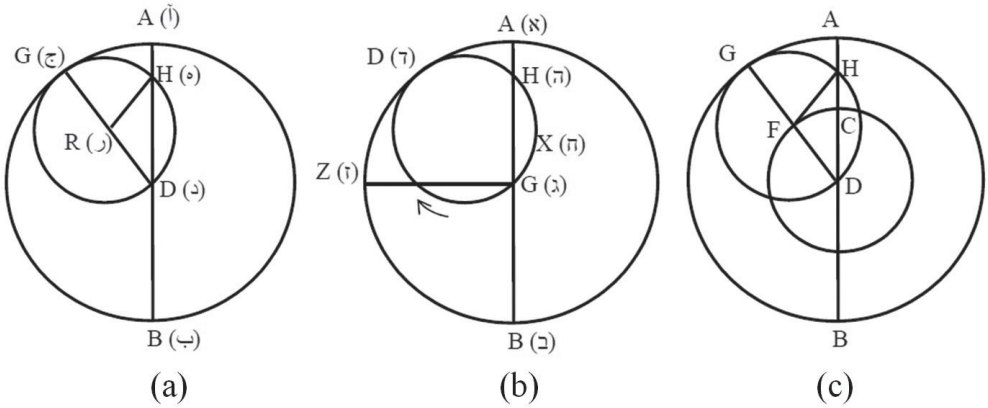


Fig. 6. (a) The Ṭūsī-couple. The small sphere is observed to rotate on the larger sphere, while the point A oscillates along the straight line AB. Hartner has made comparisons of the symbols representing the circles in Ṭūsī's original Arabic text, which are indicated by brackets. (b) Abner of Burgos is credited with the description of the geometrical properties of the Tusî-couple. (c) The model utilised by Copernicus to elucidate the motion of the planets.[32]

However, Blåsjö contends that this perspective did not originate from the transmission of Oresme's ideas; rather, it was discovered independently by Oresme during the course of his own research.[33] Blåsjö's reduction of the Ṭūsī-couple to a simple model is fundamentally based on the supposition that Copernicus also discovered this model independently. However, it is important to acknowledge the role of historical and cultural contexts in the development of this model. The model in question originated from a 500-year-old tradition of Ptolemaic criticism within the Islamic scientific community, a tradition that had not been proposed in Europe for 1,500 years. The Maragha school, in particular, focused on the adaptation of mathematics to physical reality. In this particular context, the Ṭūsī-couple was developed to address the physical inconsistencies inherent in the Ptolemaic model of the universe. Consequently, the Ṭūsī-couple may be regarded less as an isolated model and more as a solution to an epistemological crisis. The simplicity of the model is indicative not of its ordinariness, but of its elegance in the resolution of complex problems. The employment of the Ṭūsī-couple by Copernicus cannot be attributed merely to happenstance; with the resurgence of analogous issues on the European continent, it serves as an indication of the interactive and universal value of scientific knowledge.

This perspective does not detract from the originality of Copernicus's contributions, nor does it necessitate a rejection of the significant advancements and intellectual merit of Arabic astronomy. In this particular context, Michal Kokowski posits that scientific progress is achieved through scientific (r)evolutions, a process he defines as 'dwarfs standing upon the

[31] Kren 1971; Nosonovsky 2018, p. 26–27.
[32] Nosonovsky 2018, Fig.1, p. 27.
[33] Blåsjö 2018, p. 490.

shoulders of giants'. This approach enables scientists to develop a new theory without being drawn into an arrogant monologue that rejects all established science.[34]

One of the most striking features of the Ṭūsī-couple is that certain spheres direct the motion of other spheres. But why did Ṭūsī, like Copernicus, not authorise the motion of the Earth? This tendency is indicative of Ṭūsī's characteristic approach, which was that of a reformer rather than a revolutionary. Consequently, Ṭūsī remained within the confines of Ptolemy's theoretical framework. However, Ṭūsī's work demonstrated the limitations of the Ptolemaic system, both mathematically and physically, ultimately paving the way for the transformation of classical cosmology.[35]

3. Conclusion and Evaluation

The development of the Ṭūsī-couple and its reappearance a few centuries later has been traced, and the assumption that this couple was independently discovered by Copernicus has been identified as a consequence of the Eurocentric historical narrative, due to the fact that most histories of Europe, both medieval and early modern, have been written from a Eurocentric perspective. In addition, the channels through which this couple was brought to Europe have been traced. The technical terms and approaches employed by Copernicus in his treatise *'De Revolutionibus Orbium Coelestium*' demonstrate his familiarity with the Islamic astronomical tradition, particularly the utilisation of comets to challenge the veracity of Ptolemy's Almagest observations, a practice shared with Ṭūsī. The treatment of comets as an astronomical problem by Copernicus is indicative of the significance attributed to this subject in the Islamic world. In contrast, in pre-Copernican Europe, comets were predominantly associated with mystical and religious interpretations. Within the Islamic world, the movements of comets served as compelling evidence in debates concerning the rotational movement of the Earth. The absence of such evidence in Europe serves to reinforce the argument that Copernicus was significantly influenced by Islamic astronomy.

By contrast, Ptolemy's equant — a device that preserved uniform motion only around a physically unmotivated eccentric point — was treated by Islamic astronomers, Ṭūsī foremost among them, not as grounds for abandoning the Aristotelian principle of uniform circular motion but as a problem demanding its restoration through mathematically equivalent yet physically consistent alternatives. The Ṭūsī-couple stands as the most sophisticated vindication of this commitment: a demonstration that the complex appearances of planetary motion could be reconstructed entirely from combinations of uniform circular motions, thereby reproducing Ptolemaic kinematics without the cosmological compromise that the equant entailed. The mathematical resources subsequently deployed by Copernicus in his heliocentric construction had thus already been developed — and extensively debated — within the Islamic astronomical tradition. Given the challenges associated with observational verification during his era, Ṭūsī developed a mathematical model, known as the Ṭūsī-couple, to facilitate the understanding of these complex motions. It took Ṭūsī approximately fifteen years to develop a mathematical model that produced a curvilinear oscillation, thus demonstrating that scientific discoveries involve a gradual process. The Ṭūsī-couple's development — refined over fifteen years through successive versions addressing distinct astronomical problems — indicates that the model was not a simple conceptual step susceptible to independent rediscovery, but the product of a sustained intellectual tradition. This does not rule out independent discovery in principle; the history of science furnishes documented cases of convergent discovery. It does, however, raise the evidential bar: absent a parallel tradition of systematic Ptolemaic critique in pre-Copernican Europe, and given the documented transmission routes through Byzantium and the scholarly

[34] Kokowski 2012, pp. 57–58, 62.
[35] Ragep 2022, pp. 133–134.

milieux of Cracow and the Italian centres (Bologna, Padua, Ferrara), the transmission hypothesis accounts more economically for the available evidence. Instead of the Ṭūsī-couple being discovered independently, it is more probable that Copernicus obtained this knowledge through indirect channels of transmission that emerged during the evolutionary process of scientific model development. Consequently, it is hypothesised that the reflection of the Ṭūsī-couple on Copernicus's model was achieved through a cultural transmission, and the transmission channels through which this transmission occurred have been elucidated. However, the Eurocentric bias inherent in the historical account of this transmission has led to the neglect of cultural interaction networks.

The influence of the Maraga School on Copernicus's astronomical studies shows many similarities with the Ṭūsī-couple and the system of concentric spheres. This interaction took place through personal contacts, translations and cultural interaction. Especially the fact that the Ṭūsī-couple was known in Cracow and Rome in the pre-Copernican period is an important factor that facilitated Copernicus's access to this information. This interaction provides compelling evidence that scientific knowledge disseminates to diverse geographical regions through cultural transmission. The study demonstrates that the work of Copernicus was not entirely original and presents concrete data on how it was influenced by scientific developments in the Islamic world. It is evident that the Ṭūsī-couple epitomised the zenith of a millennial tradition of critical inquiry in Islamic astronomy. The fact that Copernicus produced solutions to the same problems that were equivalent to those previously identified is, in fact, the most concrete evidence of his alignment with this critical tradition. In this sense, the couple represents the culmination of a five-hundred-year tradition of constructive Ptolemaic criticism the project of kinematically principled model-building that Copernicus inherited. It is ultimately Copernicus who is recognised as the figure who inherited the legacy of this reformist critical tradition and ignited the torch of the scientific revolution.[36]

4. Declaration of competing interest

The author declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Bibliography

Bala, Arun 2016: The Scientific Revolution and the Transmission Problem. *Confluence: Journal of World Philosophies* 4, pp. 62–80. DOI: 10.5771/9783495468043-62.

Blåsjö, Viktor 2018: A Rebuttal of Recent Arguments for Maragha Influence on Copernicus. *Studia Historiae Scientiarum* 17, pp. 479–497. DOI: 10.4467/2543702XSHS.18.017.9337.

Copernicus, Nicolaus. *De Revolutionibus Orbium Coelestium*. Biblioteka Jagiellońska, Jagiellonian University, Kraków, Poland: Libri VI [Autograph manuscript] (MS 211), n.d.

Hartner, Willy 1973: Copernicus, the Man, the Work, and Its History. *Proceedings of the American Philosophical Society* 117(6), pp. 412–422. Stable URL: https://www.jstor.org/stable/986460.

Kennedy, Edward S. 1966: Late Medieval Planetary Theory'. *Isis* 57(3), pp. 365–378. Stable URL: https://www.jstor.org/stable/228366?seq=3.

[36] Kokowski 2004, pp. 65–79; 2012, pp. 57–58, 62. The essence of the Ṭūsī pair is the replacement of the Prolemean sphere/equal circle in geocentric comology with a system of spherical or circular uniform motions, in accordance with the Platonic-Aristotelian axiom of uniform circular motions. Copernicus did the same in his geokinetic cosmology. Both in Ṭūsī's and in Copernicus's constructions, equants still exist, but they become only a secondary effect, only a "phenomenon". Both Ṭūsī and Copernicus brought about scientific (r)evolutions, but the one achieved by Copernicus was more significant, as it led to the further, fruitful development of astronomy and physics. At the heart of these (r)evolutions were generalized correspondence principles.

Kokowski, Michal 2004: *Copernicus's Originality: Towards Integration of Contemporary Copernican Studies*. Warsaw-Cracow: Wydawnictwa IHN PAN, 2004, pp. XVI+340.

Kokowski, Michal 2012: Copernicus, Arabic Science, and the Scientific (R)Evolution. [In:] *Asia, Europe, and the Emergence of Modern Science*, edited by Arun Bala, pp. 55–72. DOI: https://doi.org/10.1057/9781137031730_4.

Kren, Claudia 1971: The Rolling Device of Naṣir al-Dīn al-Ṭūsī in the De spera of Nicole Oresme? *Isis* 62(4), pp. 490–498. Stable URL: https://www.jstor.org/stable/229819.

Lindberg, Davic C. 1976: *Theories of Vision from Al-Kindi to Kepler*. Edited by Allen G. Debus. Chicago-London: The University of Chicago Press.

Nosonovsky, Michael 2018: Abner of Burgos: The Missing Link between Nasir Al-Din Al-Tusi and Nicolaus Copernicus? *Zutot* 15(1), pp. 25–30. DOI: 10.1163/18750214-12151070.

Ragep, Jamil 2022: *Islamic Astronomy and Copernicus*. Ankara: Turkish Academy of Sciences Publications.

Sezgin, Fuat 2007: *Science and Technique in Islam*. 3rd edition. Vol. I. Ankara: Türkiye Academy of Sciences Publications.

Swerdlow, Noel M.; Neugebauer, Otto 1984: *Mathematical Astronomy in Copernicus's De Revolutionibus*. Salisbury, England: Springer Science+Business Media, LLC.